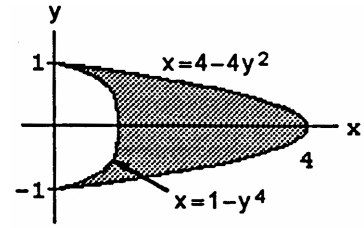
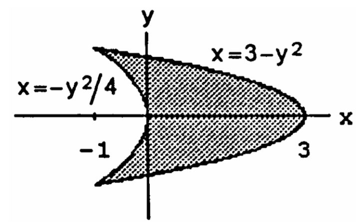
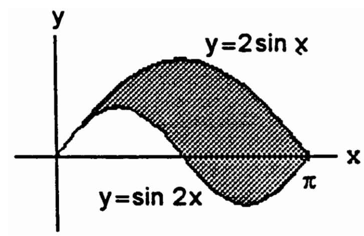
61. Limits of integration: $x = 4 - 4y^2$ and $x = 1 - y^4$
$\Rightarrow\ 4 - 4y^2 = 1 - y^4\ \Rightarrow\ y^4 - 4y^2 + 3 = 0$
$\Rightarrow\ \left(y - \sqrt{3}\right)\left(y + \sqrt{3}\right)(y-1)(y+1) = 0\ \Rightarrow\ c = -1$
and $d = 1$ since $x \geq 0$; $f(y) - g(y) = (4 - 4y^2) - (1 - y^4)$
$= 3 - 4y^2 + y^4\ \Rightarrow\ A = \int_{-1}^{1}(3 - 4y^2 + y^4)\,dy$
$= \left[3y - \frac{4y^3}{3} + \frac{y^5}{5}\right]_{-1}^{1} = 2\left(3 - \frac{4}{3} + \frac{1}{5}\right) = \frac{56}{15}$



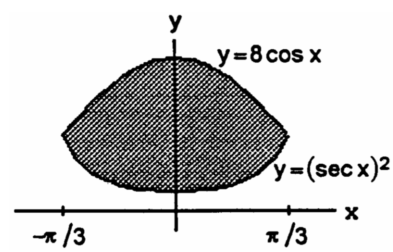
62. Limits of integration: $x = 3 - y^2$ and $x = -\frac{y^2}{4}$
$\Rightarrow\ 3 - y^2 = -\frac{y^2}{4}\ \Rightarrow\ \frac{3y^2}{4} - 3 = 0\ \Rightarrow\ \frac{3}{4}(y-2)(y+2) = 0$
$\Rightarrow\ c = -2$ and $d = 2$; $f(y) - g(y) = (3 - y^2) - \left(\frac{-y^2}{4}\right)$
$= 3\left(1 - \frac{y^2}{4}\right)\ \Rightarrow\ A = 3\int_{-2}^{2}\left(1 - \frac{y^2}{4}\right)dy = 3\left[y - \frac{y^3}{12}\right]_{-2}^{2}$
$= 3\left[\left(2 - \frac{8}{12}\right) - \left(-2 + \frac{8}{12}\right)\right] = 3\left(4 - \frac{16}{12}\right) = 12 - 4 = 8$



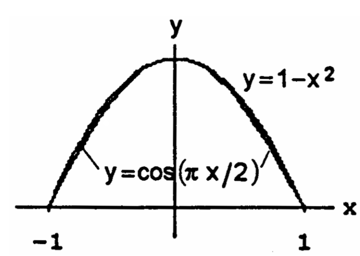
63. $a = 0,\ b = \pi$; $f(x) - g(x) = 2\sin x - \sin 2x$
$\Rightarrow\ A = \int_0^{\pi}(2\sin x - \sin 2x)\,dx = \left[-2\cos x + \frac{\cos 2x}{2}\right]_0^{\pi}$
$= \left[-2(-1) + \frac{1}{2}\right] - \left(-2 \cdot 1 + \frac{1}{2}\right) = 4$



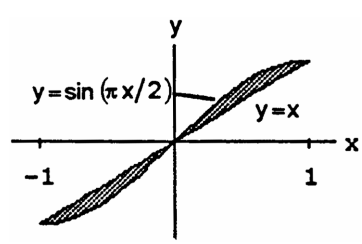
64. $a = -\frac{\pi}{3},\ b = \frac{\pi}{3}$; $f(x) - g(x) = 8\cos x - \sec^2 x$
$\Rightarrow\ A = \int_{-\pi/3}^{\pi/3}(8\cos x - \sec^2 x)\,dx = [8\sin x - \tan x]_{-\pi/3}^{\pi/3}$
$= \left(8 \cdot \frac{\sqrt{3}}{2} - \sqrt{3}\right) - \left(-8 \cdot \frac{\sqrt{3}}{2} + \sqrt{3}\right) = 6\sqrt{3}$



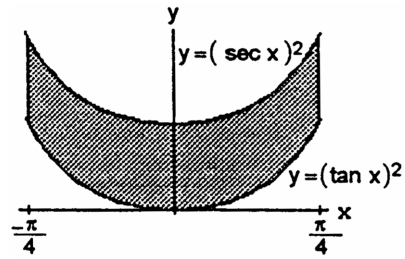
65. $a = -1,\ b = 1$; $f(x) - g(x) = (1 - x^2) - \cos\left(\frac{\pi x}{2}\right)$
$\Rightarrow\ A = \int_{-1}^{1}\left[1 - x^2 - \cos\left(\frac{\pi x}{2}\right)\right]dx = \left[x - \frac{x^3}{3} - \frac{2}{\pi}\sin\left(\frac{\pi x}{2}\right)\right]_{-1}^{1}$
$= \left(1 - \frac{1}{3} - \frac{2}{\pi}\right) - \left(-1 + \frac{1}{3} + \frac{2}{\pi}\right) = 2\left(\frac{2}{3} - \frac{2}{\pi}\right) = \frac{4}{3} - \frac{4}{\pi}$



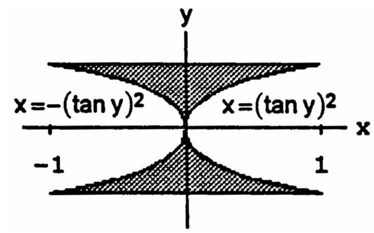
66. $A = A1 + A2$
$a_1 = -1,\ b_1 = 0$ and $a_2 = 0,\ b_2 = 1$;
$f_1(x) - g_1(x) = x - \sin\left(\frac{\pi x}{2}\right)$ and $f_2(x) - g_2(x) = \sin\left(\frac{\pi x}{2}\right) - x$
$\Rightarrow$ by symmetry about the origin,
$A_1 + A_2 = 2A_1\ \Rightarrow\ A = 2\int_0^1\left[\sin\left(\frac{\pi x}{2}\right) - x\right]dx$
$= 2\left[-\frac{2}{\pi}\cos\left(\frac{\pi x}{2}\right) - \frac{x^2}{2}\right]_0^1 = 2\left[\left(-\frac{2}{\pi} \cdot 0 - \frac{1}{2}\right) - \left(-\frac{2}{\pi} \cdot 1 - 0\right)\right]$
$= 2\left(\frac{2}{\pi} - \frac{1}{2}\right) = 2\left(\frac{4-\pi}{2\pi}\right) = \frac{4-\pi}{\pi}$



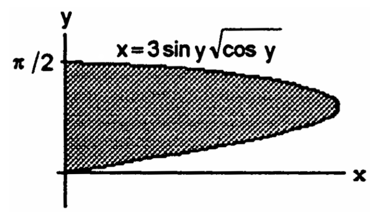
67. $a = -\frac{\pi}{4}$, $b = \frac{\pi}{4}$; $f(x) - g(x) = \sec^2 x - \tan^2 x$
$\Rightarrow A = \int_{-\pi/4}^{\pi/4} (\sec^2 x - \tan^2 x)\, dx$
$= \int_{-\pi/4}^{\pi/4} [\sec^2 x - (\sec^2 x - 1)]\, dx$
$= \int_{-\pi/4}^{\pi/4} 1 \cdot dx = [x]_{-\pi/4}^{\pi/4} = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$



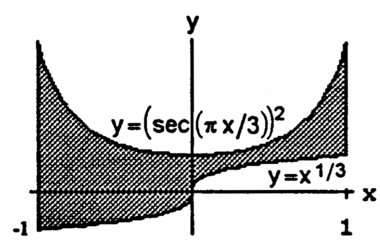
68. $c = -\frac{\pi}{4}$, $d = \frac{\pi}{4}$; $f(y) - g(y) = \tan^2 y - (-\tan^2 y) = 2\tan^2 y$
$= 2(\sec^2 y - 1) \Rightarrow A = \int_{-\pi/4}^{\pi/4} 2(\sec^2 y - 1)\, dy$
$= 2[\tan y - y]_{-\pi/4}^{\pi/4} = 2\left[\left(1 - \frac{\pi}{4}\right) - \left(-1 + \frac{\pi}{4}\right)\right]$
$= 4\left(1 - \frac{\pi}{4}\right) = 4 - \pi$



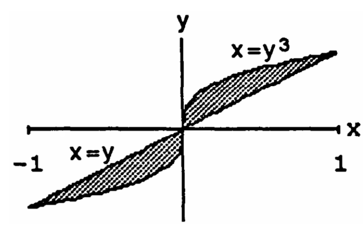
69. $c = 0$, $d = \frac{\pi}{2}$; $f(y) - g(y) = 3\sin y\sqrt{\cos y} - 0 = 3\sin y\sqrt{\cos y}$
$\Rightarrow A = 3\int_0^{\pi/2} \sin y\sqrt{\cos y}\, dy = -3\left[\frac{2}{3}(\cos y)^{3/2}\right]_0^{\pi/2}$
$= -2(0 - 1) = 2$



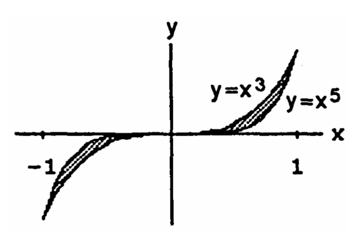
70. $a = -1$, $b = 1$; $f(x) - g(x) = \sec^2\left(\frac{\pi x}{3}\right) - x^{1/3}$
$\Rightarrow A = \int_{-1}^{1} \left[\sec^2\left(\frac{\pi x}{3}\right) - x^{1/3}\right] dx = \left[\frac{3}{\pi}\tan\left(\frac{\pi x}{3}\right) - \frac{3}{4}x^{4/3}\right]_{-1}^{1}$
$= \left(\frac{3}{\pi}\sqrt{3} - \frac{3}{4}\right) - \left[\frac{3}{\pi}\left(-\sqrt{3}\right) - \frac{3}{4}\right] = \frac{6\sqrt{3}}{\pi}$



71. $A = A_1 + A_2$
Limits of integration: $x = y^3$ and $x = y \Rightarrow y = y^3$
$\Rightarrow y^3 - y = 0 \Rightarrow y(y-1)(y+1) = 0 \Rightarrow c_1 = -1, d_1 = 0$
and $c_2 = 0$, $d_2 = 1$; $f_1(y) - g_1(y) = y^3 - y$ and
$f_2(y) - g_2(y) = y - y^3 \Rightarrow$ by symmetry about the origin,
$A_1 + A_2 = 2A_2 \Rightarrow A = 2\int_0^1 (y - y^3)\, dy = 2\left[\frac{y^2}{2} - \frac{y^4}{4}\right]_0^1$
$= 2\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{1}{2}$



72. $A = A_1 + A_2$
Limits of integration: $y = x^3$ and $y = x^5 \Rightarrow x^3 = x^5$
$\Rightarrow x^5 - x^3 = 0 \Rightarrow x^3(x-1)(x+1) = 0 \Rightarrow a_1 = -1, b_1 = 0$
and $a_2 = 0$, $b_2 = 1$; $f_1(x) - g_1(x) = x^3 - x^5$ and
$f_2(x) - g_2(x) = x^5 - x^3 \Rightarrow$ by symmetry about the origin,
$A_1 + A_2 = 2A_2 \Rightarrow A = 2\int_0^1 (x^3 - x^5)\, dx = 2\left[\frac{x^4}{4} - \frac{x^6}{6}\right]_0^1$
$= 2\left(\frac{1}{4} - \frac{1}{6}\right) = \frac{1}{6}$



73. $A = A_1 + A_2$

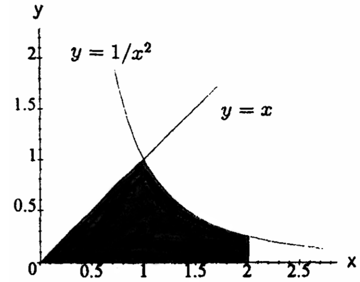
Limits of integration: $y = x$ and $y = \frac{1}{x^2} \Rightarrow x = \frac{1}{x^2}, x \neq 0$

$\Rightarrow x^3 = 1 \Rightarrow x = 1$, $f_1(x) - g_1(x) = x - 0 = x$

$\Rightarrow A_1 = \int_0^1 x\,dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}$; $f_2(x) - g_2(x) = \frac{1}{x^2} - 0$

$= x^{-2} \Rightarrow A_2 = \int_1^2 x^{-2}\,dx = \left[\frac{-1}{x}\right]_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$;

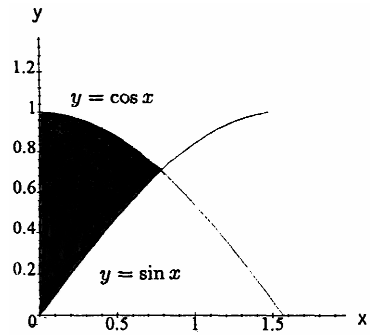
$A = A_1 + A_2 = \frac{1}{2} + \frac{1}{2} = 1$



74. Limits of integration: $\sin x = \cos x \Rightarrow x = \frac{\pi}{4} \Rightarrow a = 0$ and $b = \frac{\pi}{4}$; $f(x) - g(x) = \cos x - \sin x$

$\Rightarrow A = \int_0^{\pi/4} (\cos x - \sin x)\,dx = [\sin x + \cos x]_0^{\pi/4}$

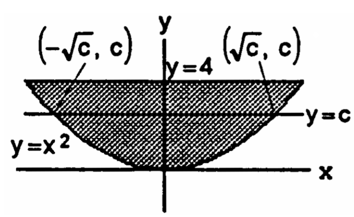
$= \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - (0 + 1) = \sqrt{2} - 1$



75. (a) The coordinates of the points of intersection of the line and parabola are $c = x^2 \Rightarrow x = \pm\sqrt{c}$ and $y = c$

(b) $f(y) - g(y) = \sqrt{y} - \left(-\sqrt{y}\right) = 2\sqrt{y} \Rightarrow$ the area of the lower section is, $A_L = \int_0^c [f(y) - g(y)]\,dy$

$= 2\int_0^c \sqrt{y}\,dy = 2\left[\frac{2}{3}y^{3/2}\right]_0^c = \frac{4}{3}c^{3/2}$. The area of the entire shaded region can be found by setting $c = 4$: $A = \left(\frac{4}{3}\right)4^{3/2} = \frac{4\cdot 8}{3} = \frac{32}{3}$. Since we want c to divide the region into subsections of equal area we have $A = 2A_L \Rightarrow \frac{32}{3} = 2\left(\frac{4}{3}c^{3/2}\right) \Rightarrow c = 4^{2/3}$



(c) $f(x) - g(x) = c - x^2 \Rightarrow A_L = \int_{-\sqrt{c}}^{\sqrt{c}} [f(x) - g(x)]\,dx = \int_{-\sqrt{c}}^{\sqrt{c}} (c - x^2)\,dx = \left[cx - \frac{x^3}{3}\right]_{-\sqrt{c}}^{\sqrt{c}} = 2\left[c^{3/2} - \frac{c^{3/2}}{3}\right]$

$= \frac{4}{3}c^{3/2}$. Again, the area of the whole shaded region can be found by setting $c = 4 \Rightarrow A = \frac{32}{3}$. From the condition $A = 2A_L$, we get $\frac{4}{3}c^{3/2} = \frac{32}{3} \Rightarrow c = 4^{2/3}$ as in part (b).

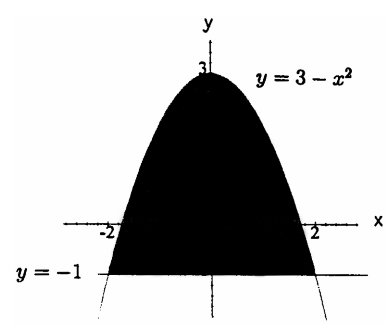
76. (a) Limits of integration: $y = 3 - x^2$ and $y = -1$

$\Rightarrow 3 - x^2 = -1 \Rightarrow x^2 = 4 \Rightarrow a = -2$ and $b = 2$;

$f(x) - g(x) = (3 - x^2) - (-1) = 4 - x^2$

$\Rightarrow A = \int_{-2}^{2} (4 - x^2)\,dx = \left[4x - \frac{x^3}{3}\right]_{-2}^{2}$

$= \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right) = 16 - \frac{16}{3} = \frac{32}{3}$



(b) Limits of integration: let $x = 0$ in $y = 3 - x^2$

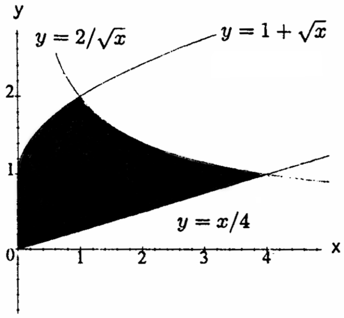
$\Rightarrow y = 3$; $f(y) - g(y) = \sqrt{3 - y} - \left(-\sqrt{3 - y}\right)$

$= 2(3 - y)^{1/2}$

$\Rightarrow A = 2\int_{-1}^{3} (3 - y)^{1/2}\,dy = -2\int_{-1}^{3} (3 - y)^{1/2}(-1)\,dy = (-2)\left[\frac{2(3 - y)^{3/2}}{3}\right]_{-1}^{3} = \left(-\frac{4}{3}\right)\left[0 - (3 + 1)^{3/2}\right]$

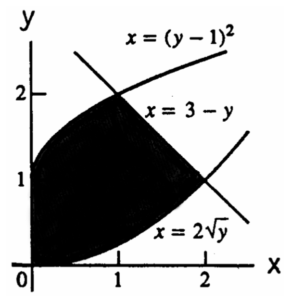
$= \left(\frac{4}{3}\right)(8) = \frac{32}{3}$

77. Limits of integration: $y = 1 + \sqrt{x}$ and $y = \frac{2}{\sqrt{x}}$
$\Rightarrow\ 1 + \sqrt{x} = \frac{2}{\sqrt{x}},\ x \neq 0\ \Rightarrow\ \sqrt{x} + x = 2\ \Rightarrow\ x = (2 - x)^2$
$\Rightarrow\ x = 4 - 4x + x^2\ \Rightarrow\ x^2 - 5x + 4 = 0$
$\Rightarrow\ (x-4)(x-1) = 0\ \Rightarrow\ x = 1, 4$ (but $x = 4$ does not satisfy the equation); $y = \frac{2}{\sqrt{x}}$ and $y = \frac{x}{4}\ \Rightarrow\ \frac{2}{\sqrt{x}} = \frac{x}{4}$
$\Rightarrow\ 8 = x\sqrt{x}\ \Rightarrow\ 64 = x^3\ \Rightarrow\ x = 4.$



Therefore, AREA $= A_1 + A_2$: $f_1(x) - g_1(x) = \left(1 + x^{1/2}\right) - \frac{x}{4}$
$\Rightarrow\ A_1 = \int_0^1 \left(1 + x^{1/2} - \frac{x}{4}\right) dx = \left[x + \frac{2}{3}x^{3/2} - \frac{x^2}{8}\right]_0^1$
$= \left(1 + \frac{2}{3} - \frac{1}{8}\right) - 0 = \frac{37}{24}$; $f_2(x) - g_2(x) = 2x^{-1/2} - \frac{x}{4}\ \Rightarrow\ A_2 = \int_1^4 \left(2x^{-1/2} - \frac{x}{4}\right) dx\ = \left[4x^{1/2} - \frac{x^2}{8}\right]_1^4$
$= \left(4 \cdot 2 - \frac{16}{8}\right) - \left(4 - \frac{1}{8}\right) = 4 - \frac{15}{8} = \frac{17}{8}$; Therefore, AREA $= A_1 + A_2 = \frac{37}{24} + \frac{17}{8} = \frac{37+51}{24} = \frac{88}{24} = \frac{11}{3}$

78. Limits of integration: $(y-1)^2 = 3 - y\ \Rightarrow\ y^2 - 2y + 1$
$= 3 - y\ \Rightarrow\ y^2 - y - 2 = 0\ \Rightarrow\ (y-2)(y+1) = 0$
$\Rightarrow\ y = 2$ since $y > 0$; also, $2\sqrt{y} = 3 - y$
$\Rightarrow\ 4y = 9 - 6y + y^2\ \Rightarrow\ y^2 - 10y + 9 = 0$
$\Rightarrow\ (y-9)(y-1) = 0\ \Rightarrow\ y = 1$ since $y = 9$ does not satisfy the equation;
AREA $= A_1 + A_2$
$f_1(y) - g_1(y) = 2\sqrt{y} - 0 = 2y^{1/2}$



$\Rightarrow\ A_1 = 2\int_0^1 y^{1/2}\, dy = 2\left[\frac{2y^{3/2}}{3}\right]_0^1 = \frac{4}{3}$; $f_2(y) - g_2(y) = (3 - y) - (y-1)^2$
$\Rightarrow\ A_2 = \int_1^2 \left[3 - y - (y-1)^2\right] dy = \left[3y - \frac{1}{2}y^2 - \frac{1}{3}(y-1)^3\right]_1^2 = \left(6 - 2 - \frac{1}{3}\right) - \left(3 - \frac{1}{2} + 0\right) = 1 - \frac{1}{3} + \frac{1}{2} = \frac{7}{6}$;
Therefore, $A_1 + A_2 = \frac{4}{3} + \frac{7}{6} = \frac{15}{6} = \frac{5}{2}$

79. Area between parabola and $y = a^2$: $A = 2\int_0^a \left(a^2 - x^2\right) dx = 2\left[a^2x - \frac{1}{3}x^3\right]_0^a = 2\left(a^3 - \frac{a^3}{3}\right) - 0 = \frac{4a^3}{3}$;
Area of triangle AOC: $\frac{1}{2}(2a)\left(a^2\right) = a^3$; limit of ratio $= \lim\limits_{a \to 0^+} \frac{a^3}{\left(\frac{4a^3}{3}\right)} = \frac{3}{4}$ which is independent of a.

80. $A = \int_a^b 2f(x)\,dx - \int_a^b f(x)\,dx = 2\int_a^b f(x)\,dx - \int_a^b f(x)\,dx = \int_a^b f(x)\,dx = 4$

81. The lower boundary of the region is the line through the points $(z,\ 1 - z^2)$ and $\left(z + 1,\ 1 - (z+1)^2\right)$. The equation of this line is $y - (1 - z^2) = \frac{(1-(z+1)^2) - (1-z^2)}{z+1-z}(x - 1) = -(2z+1)(x-1) \Rightarrow y = -(2z+1)x + (z^2 + z + 1)$.
The area of theregion is given by $\int_z^{z+1} \left((1 - x^2) - (-(2z+1)x + (z^2 + z + 1))\right)dy$
$= \int_z^{z+1} \left(-x^2 + (2z+1)x - z^2 - z\right)dy = \left[-\frac{1}{3}x^3 + \frac{1}{2}(2z+1)x^2 - (z^2+z)x\right]_z^{z+1}$
$= \left(-\frac{1}{3}(z+1)^3 + \frac{1}{2}(2z+1)(z+1)^2 - (z^2+z)(z+1)\right) - \left(-\frac{1}{3}z^3 + \frac{1}{2}(2z+1)z^2 - (z^2+z)z\right) = \frac{1}{6}$. No matter where we choose z, the area of the region bounded by $y = 1 - x^2$ and the line through the points $(z,\ 1 - z^2)$ and $\left(z+1,\ 1 - (z+1)^2\right)$ is always $\frac{1}{6}$.

82. It is sometimes true. It is true if $f(x) \geq g(x)$ for all x between a and b. Otherwise it is false. If the graph of f lies below the graph of g for a portion of the interval of integration, the integral over that portion will be negative and the integral over $[a, b]$ will be less than the area between the curves (see Exercise 71).

83. Let $u = 2x \Rightarrow du = 2\,dx \Rightarrow \frac{1}{2}\,du = dx$; $x = 1 \Rightarrow u = 2$, $x = 3 \Rightarrow u = 6$

$$\int_1^3 \frac{\sin 2x}{x}\,dx = \int_2^6 \frac{\sin u}{\left(\frac{u}{2}\right)}\left(\frac{1}{2}\,du\right) = \int_2^6 \frac{\sin u}{u}\,du = [F(u)]_2^6 = F(6) - F(2)$$

84. Let $u = 1 - x \Rightarrow du = -dx \Rightarrow -du = dx$; $x = 0 \Rightarrow u = 1$, $x = 1 \Rightarrow u = 0$

$$\int_0^1 f(1-x)\,dx = \int_1^0 f(u)\,(-du) = -\int_1^0 f(u)\,du = \int_0^1 f(u)\,du = \int_0^1 f(x)\,dx$$

85. (a) Let $u = -x \Rightarrow du = -dx$; $x = -1 \Rightarrow u = 1$, $x = 0 \Rightarrow u = 0$

f odd $\Rightarrow f(-x) = -f(x)$. Then $\int_{-1}^0 f(x)\,dx = \int_1^0 f(-u)\,(-du) = \int_1^0 -f(u)\,(-du) = \int_1^0 f(u)\,du = -\int_0^1 f(u)\,du$ $= -3$

(b) Let $u = -x \Rightarrow du = -dx$; $x = -1 \Rightarrow u = 1$, $x = 0 \Rightarrow u = 0$

f even $\Rightarrow f(-x) = f(x)$. Then $\int_{-1}^0 f(x)\,dx = \int_1^0 f(-u)\,(-du) = -\int_1^0 f(u)\,du = \int_0^1 f(u)\,du = 3$

86. (a) Consider $\int_{-a}^0 f(x)\,dx$ when f is odd. Let $u = -x \Rightarrow du = -dx \Rightarrow -du = dx$ and $x = -a \Rightarrow u = a$ and $x = 0$ $\Rightarrow u = 0$. Thus $\int_{-a}^0 f(x)\,dx = \int_a^0 -f(-u)\,du = \int_a^0 f(u)\,du = -\int_0^a f(u)\,du = -\int_0^a f(x)\,dx$.

Thus $\int_{-a}^a f(x)\,dx = \int_{-a}^0 f(x)\,dx + \int_0^a f(x)\,dx = -\int_0^a f(x)\,dx + \int_0^a f(x)\,dx = 0$.

(b) $\int_{-\pi/2}^{\pi/2} \sin x\,dx = [-\cos x]_{-\pi/2}^{\pi/2} = -\cos\left(\frac{\pi}{2}\right) + \cos\left(-\frac{\pi}{2}\right) = 0 + 0 = 0.$

87. Let $u = a - x \Rightarrow du = -dx$; $x = 0 \Rightarrow u = a$, $x = a \Rightarrow u = 0$

$$I = \int_0^a \frac{f(x)\,dx}{f(x)+f(a-x)} = \int_a^0 \frac{f(a-u)}{f(a-u)+f(u)}\,(-du) = \int_0^a \frac{f(a-u)\,du}{f(u)+f(a-u)} = \int_0^a \frac{f(a-x)\,dx}{f(x)+f(a-x)}$$

$$\Rightarrow I + I = \int_0^a \frac{f(x)\,dx}{f(x)+f(a-x)} + \int_0^a \frac{f(a-x)\,dx}{f(x)+f(a-x)} = \int_0^a \frac{f(x)+f(a-x)}{f(x)+f(a-x)}\,dx = \int_0^a dx = [x]_0^a = a - 0 = a.$$

Therefore, $2I = a \Rightarrow I = \frac{a}{2}$.

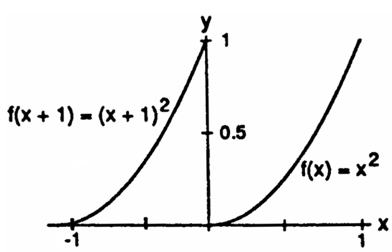
88. Let $u = \frac{xy}{t} \Rightarrow du = -\frac{xy}{t^2}\,dt \Rightarrow -\frac{t}{xy}\,du = \frac{1}{t}\,dt \Rightarrow -\frac{1}{u}\,du = \frac{1}{t}\,dt$; $t = x \Rightarrow u = y$, $t = xy \Rightarrow u = 1$. Therefore,

$$\int_x^{xy} \frac{1}{t}\,dt = \int_y^1 -\frac{1}{u}\,du = -\int_y^1 \frac{1}{u}\,du = \int_1^y \frac{1}{u}\,du = \int_1^y \frac{1}{t}\,dt$$

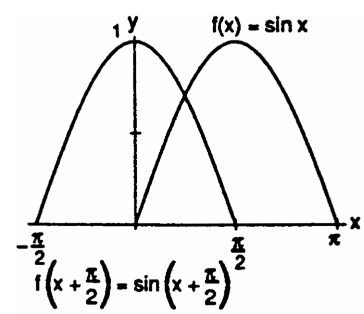
89. Let $u = x + c \Rightarrow du = dx$; $x = a - c \Rightarrow u = a$, $x = b - c \Rightarrow u = b$

$$\int_{a-c}^{b-c} f(x+c)\,dx = \int_a^b f(u)\,du = \int_a^b f(x)\,dx$$

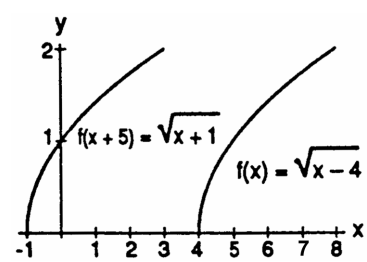
90. (a)



(b)



(c)



91-94. Example CAS commands:

Maple:

```
f := x -> x^3/3-x^2/2-2*x+1/3;
g := x -> x-1;
plot( [f(x),g(x)], x=-5..5, legend=["y = f(x)","y = g(x)"], title="#91(a) (Section 5.6)" );
q1 := [ -5, -2, 1, 4 ];                    # (b)
q2 := [seq( fsolve( f(x)=g(x), x=q1[i]..q1[i+1] ), i=1..nops(q1)-1 )];
for i from 1 to nops(q2)-1 do        # (c)
  area[i] := int( abs(f(x)-g(x)),x=q2[i]..q2[i+1] );
end do;
add( area[i], i=1..nops(q2)-1 );      # (d)
```

Mathematica: (assigned functions may vary)

```
Clear[x, f, g]
f[x_] = x^2 Cos[x]
g[x_] = x^3 − x
Plot[{f[x], g[x]}, {x, −2, 2}]
```

After examining the plots, the initial guesses for FindRoot can be determined.

```
pts = x/.Map[FindRoot[f[x]==g[x],{x, #}]&, {−1, 0, 1}]
i1=NIntegrate[f[x] − g[x], {x, pts[[1]], pts[[2]]}]
i2=NIntegrate[f[x] − g[x], {x, pts[[2]], pts[[3]]}]
i1 + i2
```

CHAPTER 5 PRACTICE EXERCISES

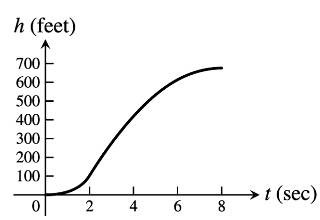
1. (a) Each time subinterval is of length $\Delta t = 0.4$ sec. The distance traveled over each subinterval, using the midpoint rule, is $\Delta h = \frac{1}{2}(v_i + v_{i+1})\Delta t$, where v_i is the velocity at the left endpoint and v_{i+1} the velocity at the right endpoint of the subinterval. We then add Δh to the height attained so far at the left endpoint v_i to arrive at the height associated with velocity v_{i+1} at the right endpoint. Using this methodology we build the following table based on the figure in the text:

t (sec)	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2	3.6	4.0	4.4	4.8	5.2	5.6	6.0
v (fps)	0	10	25	55	100	190	180	165	150	140	130	115	105	90	76	65
h (ft)	0	2	9	25	56	114	188	257	320	378	432	481	525	564	592	620.2

t (sec)	6.4	6.8	7.2	7.6	8.0
v (fps)	50	37	25	12	0
h (ft)	643.2	660.6	672	679.4	681.8

NOTE: Your table values may vary slightly from ours depending on the v-values you read from the graph. Remember that some shifting of the graph occurs in the printing process.
The total height attained is about 680 ft.

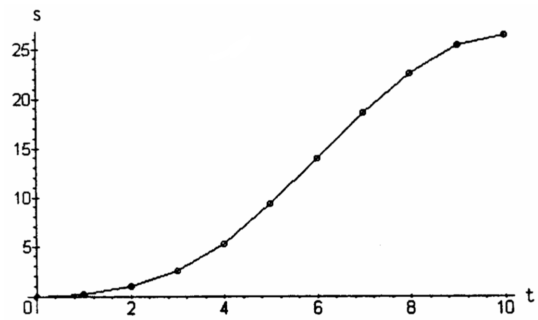
(b) The graph is based on the table in part (a).



2. (a) Each time subinterval is of length $\Delta t = 1$ sec. The distance traveled over each subinterval, using the midpoint rule, is $\Delta s = \frac{1}{2}(v_i + v_{i+1})\,\Delta t$, where v_i is the velocity at the left, and v_{i+1} the velocity at the right, endpoint of the subinterval. We then add Δs to the distance attained so far at the left endpoint v_i to arrive at the distance associated with velocity v_{i+1} at the right endpoint. Using this methodology we build the table given below based on the figure in the text, obtaining approximately 26 m for the total distance traveled:

t (sec)	0	1	2	3	4	5	6	7	8	9	10
v (m/sec)	0	0.5	1.2	2	3.4	4.5	4.8	4.5	3.5	2	0
s (m)	0	0.25	1.1	2.7	5.4	9.35	14	18.65	22.65	25.4	26.4

(b) The graph shows the distance traveled by the moving body as a function of time for $0 \le t \le 10$.



3. (a) $\sum_{k=1}^{10} \frac{a_k}{4} = \frac{1}{4}\sum_{k=1}^{10} a_k = \frac{1}{4}(-2) = -\frac{1}{2}$ (b) $\sum_{k=1}^{10} (b_k - 3a_k) = \sum_{k=1}^{10} b_k - 3\sum_{k=1}^{10} a_k = 25 - 3(-2) = 31$

(c) $\sum_{k=1}^{10} (a_k + b_k - 1) = \sum_{k=1}^{10} a_k + \sum_{k=1}^{10} b_k - \sum_{k=1}^{10} 1 = -2 + 25 - (1)(10) = 13$

(d) $\sum_{k=1}^{10} \left(\frac{5}{2} - b_k\right) = \sum_{k=1}^{10} \frac{5}{2} - \sum_{k=1}^{10} b_k = \frac{5}{2}(10) - 25 = 0$

4. (a) $\sum_{k=1}^{20} 3a_k = 3\sum_{k=1}^{20} a_k = 3(0) = 0$ (b) $\sum_{k=1}^{20} (a_k + b_k) = \sum_{k=1}^{20} a_k + \sum_{k=1}^{20} b_k = 0 + 7 = 7$

(c) $\sum_{k=1}^{20} \left(\frac{1}{2} - \frac{2b_k}{7}\right) = \sum_{k=1}^{20} \frac{1}{2} - \frac{2}{7}\sum_{k=1}^{20} b_k = \frac{1}{2}(20) - \frac{2}{7}(7) = 8$

(d) $\sum_{k=1}^{20} (a_k - 2) = \sum_{k=1}^{20} a_k - \sum_{k=1}^{20} 2 = 0 - 2(20) = -40$

5. Let $u = 2x - 1 \Rightarrow du = 2\,dx \Rightarrow \frac{1}{2}\,du = dx$; $x = 1 \Rightarrow u = 1$, $x = 5 \Rightarrow u = 9$

$\int_1^5 (2x-1)^{-1/2}\,dx = \int_1^9 u^{-1/2}\left(\frac{1}{2}\,du\right) = \left[u^{1/2}\right]_1^9 = 3 - 1 = 2$

6. Let $u = x^2 - 1 \Rightarrow du = 2x\,dx \Rightarrow \frac{1}{2}\,du = x\,dx$; $x = 1 \Rightarrow u = 0$, $x = 3 \Rightarrow u = 8$

$\int_1^3 x\,(x^2-1)^{1/3}\,dx = \int_0^8 u^{1/3}\left(\frac{1}{2}\,du\right) = \left[\frac{3}{8}u^{4/3}\right]_0^8 = \frac{3}{8}(16 - 0) = 6$

7. Let $u = \frac{x}{2} \Rightarrow 2\,du = dx$; $x = -\pi \Rightarrow u = -\frac{\pi}{2}$, $x = 0 \Rightarrow u = 0$

$\int_{-\pi}^0 \cos\left(\frac{x}{2}\right)dx = \int_{-\pi/2}^0 (\cos u)(2\,du) = [2\sin u]_{-\pi/2}^0 = 2\sin 0 - 2\sin\left(-\frac{\pi}{2}\right) = 2(0 - (-1)) = 2$

8. Let $u = \sin x \Rightarrow du = \cos x\,dx$; $x = 0 \Rightarrow u = 0$, $x = \frac{\pi}{2} \Rightarrow u = 1$

$\int_0^{\pi/2} (\sin x)(\cos x)\,dx = \int_0^1 u\,du = \left[\frac{u^2}{2}\right]_0^1 = \frac{1}{2}$

9. (a) $\int_{-2}^{2} f(x)\,dx = \frac{1}{3}\int_{-2}^{2} 3\,f(x)\,dx = \frac{1}{3}(12) = 4$ (b) $\int_{2}^{5} f(x)\,dx = \int_{-2}^{5} f(x)\,dx - \int_{-2}^{2} f(x)\,dx = 6 - 4 = 2$

(c) $\int_{5}^{-2} g(x)\,dx = -\int_{-2}^{5} g(x)\,dx = -2$ (d) $\int_{-2}^{5} (-\pi\, g(x))\,dx = -\pi \int_{-2}^{5} g(x)\,dx = -\pi(2) = -2\pi$

(e) $\int_{-2}^{5} \left(\frac{f(x)+g(x)}{5}\right) dx = \frac{1}{5}\int_{-2}^{5} f(x)\,dx + \frac{1}{5}\int_{-2}^{5} g(x)\,dx = \frac{1}{5}(6) + \frac{1}{5}(2) = \frac{8}{5}$

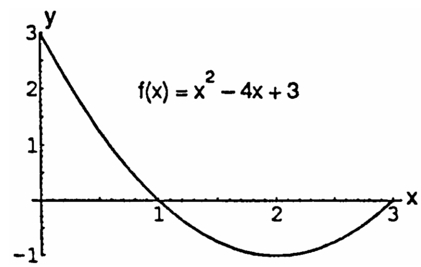
10. (a) $\int_{0}^{2} g(x)\,dx = \frac{1}{7}\int_{0}^{2} 7\,g(x)\,dx = \frac{1}{7}(7) = 1$ (b) $\int_{1}^{2} g(x)\,dx = \int_{0}^{2} g(x)\,dx - \int_{0}^{1} g(x)\,dx = 1 - 2 = -1$

(c) $\int_{2}^{0} f(x)\,dx = -\int_{0}^{2} f(x)\,dx = -\pi$ (d) $\int_{0}^{2} \sqrt{2}\, f(x)\,dx = \sqrt{2}\int_{0}^{2} f(x)\,dx = \sqrt{2}(\pi) = \pi\sqrt{2}$

(e) $\int_{0}^{2} [g(x) - 3\,f(x)]\,dx = \int_{0}^{2} g(x)\,dx - 3\int_{0}^{2} f(x)\,dx = 1 - 3\pi$

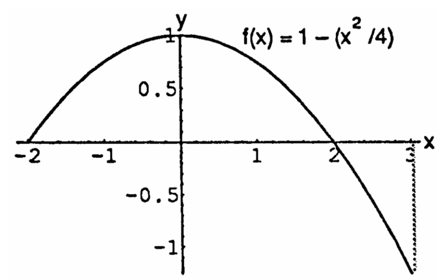
11. $x^2 - 4x + 3 = 0 \Rightarrow (x-3)(x-1) = 0 \Rightarrow x = 3$ or $x = 1$;

$$\text{Area} = \int_{0}^{1} (x^2 - 4x + 3)\,dx - \int_{1}^{3} (x^2 - 4x + 3)\,dx$$
$$= \left[\frac{x^3}{3} - 2x^2 + 3x\right]_{0}^{1} - \left[\frac{x^3}{3} - 2x^2 + 3x\right]_{1}^{3}$$
$$= \left[\left(\frac{1^3}{3} - 2(1)^2 + 3(1)\right) - 0\right]$$
$$- \left[\left(\frac{3^3}{3} - 2(3)^2 + 3(3)\right) - \left(\frac{1^3}{3} - 2(1)^2 + 3(1)\right)\right]$$
$$= \left(\frac{1}{3} + 1\right) - \left[0 - \left(\frac{1}{3} + 1\right)\right] = \frac{8}{3}$$



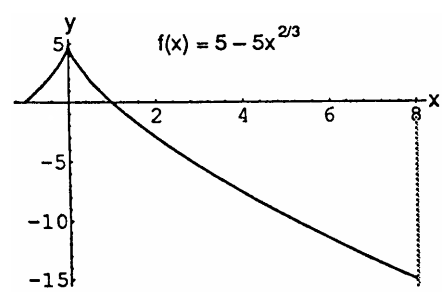
12. $1 - \frac{x^2}{4} = 0 \Rightarrow 4 - x^2 - 0 \Rightarrow x = \pm 2$;

$$\text{Area} = \int_{-2}^{2} \left(1 - \frac{x^2}{4}\right) dx - \int_{2}^{3} \left(1 - \frac{x^2}{4}\right) dx$$
$$= \left[x - \frac{x^3}{12}\right]_{-2}^{2} - \left[x - \frac{x^3}{12}\right]_{2}^{3}$$
$$= \left[\left(2 - \frac{2^3}{12}\right) - \left(-2 - \frac{(-2)^3}{12}\right)\right] - \left[\left(3 - \frac{3^3}{12}\right) - \left(2 - \frac{2^3}{12}\right)\right]$$
$$= \left[\frac{4}{3} - \left(-\frac{4}{3}\right)\right] - \left(\frac{3}{4} - \frac{4}{3}\right) = \frac{13}{4}$$



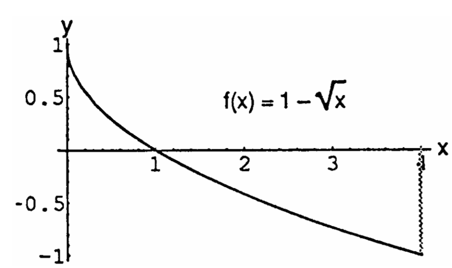
13. $5 - 5x^{2/3} = 0 \Rightarrow 1 - x^{2/3} = 0 \Rightarrow x = \pm 1$;

$$\text{Area} = \int_{-1}^{1} \left(5 - 5x^{2/3}\right) dx - \int_{1}^{8} \left(5 - 5x^{2/3}\right) dx$$
$$= \left[5x - 3x^{5/3}\right]_{-1}^{1} - \left[5x - 3x^{5/3}\right]_{1}^{8}$$
$$= \left[\left(5(1) - 3(1)^{5/3}\right) - \left(5(-1) - 3(-1)^{5/3}\right)\right]$$
$$- \left[\left(5(8) - 3(8)^{5/3}\right) - \left(5(1) - 3(1)^{5/3}\right)\right]$$
$$= [2 - (-2)] - [(40 - 96) - 2] = 62$$

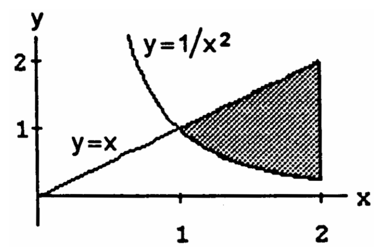


14. $1 - \sqrt{x} = 0 \Rightarrow x = 1$;

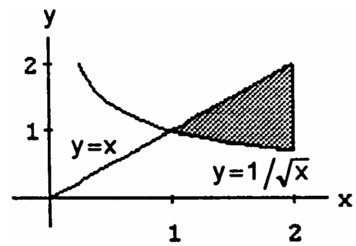
$$\text{Area} = \int_{0}^{1} \left(1 - \sqrt{x}\right) dx - \int_{1}^{4} \left(1 - \sqrt{x}\right) dx$$
$$= \left[x - \frac{2}{3}x^{3/2}\right]_{0}^{1} - \left[x - \frac{2}{3}x^{3/2}\right]_{1}^{4}$$
$$= \left[\left(1 - \frac{2}{3}(1)^{3/2}\right) - 0\right] - \left[\left(4 - \frac{2}{3}(4)^{3/2}\right) - \left(1 - \frac{2}{3}(1)^{3/2}\right)\right]$$
$$= \frac{1}{3} - \left[\left(4 - \frac{16}{3}\right) - \frac{1}{3}\right] = 2$$



15. $f(x) = x,\ g(x) = \frac{1}{x^2},\ a = 1,\ b = 2 \Rightarrow A = \int_a^b [f(x) - g(x)]\,dx$
$= \int_1^2 \left(x - \frac{1}{x^2}\right) dx = \left[\frac{x^2}{2} + \frac{1}{x}\right]_1^2 = \left(\frac{4}{2} + \frac{1}{2}\right) - \left(\frac{1}{2} + 1\right) = 1$



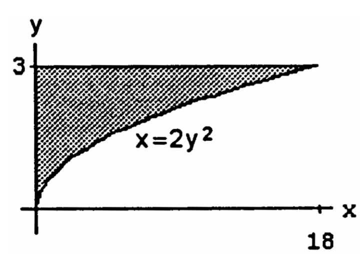
16. $f(x) = x,\ g(x) = \frac{1}{\sqrt{x}},\ a = 1,\ b = 2 \Rightarrow A = \int_a^b [f(x) - g(x)]\,dx$
$= \int_1^2 \left(x - \frac{1}{\sqrt{x}}\right) dx = \left[\frac{x^2}{2} - 2\sqrt{x}\right]_1^2$
$= \left(\frac{4}{2} - 2\sqrt{2}\right) - \left(\frac{1}{2} - 2\right) = \frac{7 - 4\sqrt{2}}{2}$



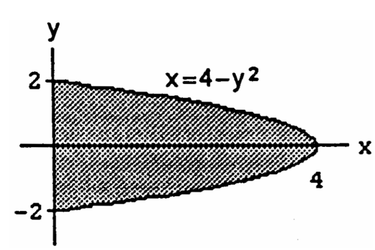
17. $f(x) = \left(1 - \sqrt{x}\right)^2,\ g(x) = 0,\ a = 0,\ b = 1 \Rightarrow A = \int_a^b [f(x) - g(x)]\,dx = \int_0^1 \left(1 - \sqrt{x}\right)^2 dx = \int_0^1 \left(1 - 2\sqrt{x} + x\right) dx$
$= \int_0^1 \left(1 - 2x^{1/2} + x\right) dx = \left[x - \frac{4}{3}x^{3/2} + \frac{x^2}{2}\right]_0^1 = 1 - \frac{4}{3} + \frac{1}{2} = \frac{1}{6}(6 - 8 + 3) = \frac{1}{6}$

18. $f(x) = \left(1 - x^3\right)^2,\ g(x) = 0,\ a = 0,\ b = 1 \Rightarrow A = \int_a^b [f(x) - g(x)]\,dx = \int_0^1 \left(1 - x^3\right)^2 dx = \int_0^1 \left(1 - 2x^3 + x^6\right) dx$
$= \left[x - \frac{x^4}{2} + \frac{x^7}{7}\right]_0^1 = 1 - \frac{1}{2} + \frac{1}{7} = \frac{9}{14}$

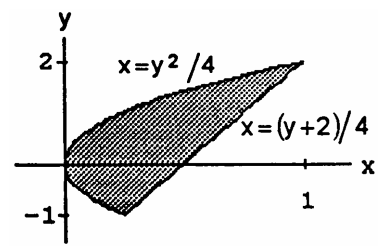
19. $f(y) = 2y^2,\ g(y) = 0,\ c = 0,\ d = 3$
$\Rightarrow A = \int_c^d [f(y) - g(y)]\,dy = \int_0^3 \left(2y^2 - 0\right) dy$
$= 2\int_0^3 y^2\,dy = \frac{2}{3}\left[y^3\right]_0^3 = 18$



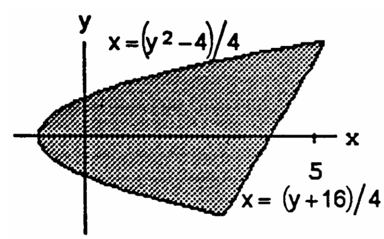
20. $f(y) = 4 - y^2,\ g(y) = 0,\ c = -2,\ d = 2$
$\Rightarrow A = \int_c^d [f(y) - g(y)]\,dy = \int_{-2}^2 \left(4 - y^2\right) dy$
$= \left[4y - \frac{y^3}{3}\right]_{-2}^2 = 2\left(8 - \frac{8}{3}\right) = \frac{32}{3}$



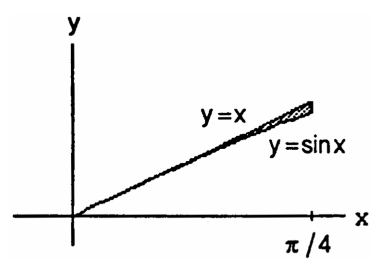
21. Let us find the intersection points: $\frac{y^2}{4} = \frac{y+2}{4}$
$\Rightarrow y^2 - y - 2 = 0 \Rightarrow (y-2)(y+1) = 0 \Rightarrow y = -1$
or $y = 2 \Rightarrow c = -1, d = 2; f(y) = \frac{y+2}{4}, g(y) = \frac{y^2}{4}$
$\Rightarrow A = \int_c^d [f(y) - g(y)]\, dy = \int_{-1}^{2} \left(\frac{y+2}{4} - \frac{y^2}{4}\right) dy$
$= \frac{1}{4}\int_{-1}^{2} (y + 2 - y^2)\, dy = \frac{1}{4}\left[\frac{y^2}{2} + 2y - \frac{y^3}{3}\right]_{-1}^{2}$
$= \frac{1}{4}\left[\left(\frac{4}{2} + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)\right] = \frac{9}{8}$



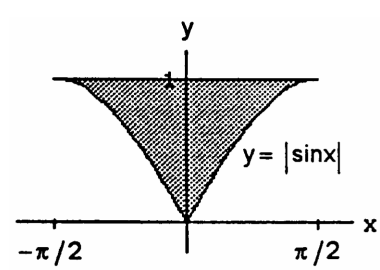
22. Let us find the intersection points: $\frac{y^2-4}{4} = \frac{y+16}{4}$
$\Rightarrow y^2 - y - 20 = 0 \Rightarrow (y-5)(y+4) = 0 \Rightarrow y = -4$
or $y = 5 \Rightarrow c = -4, d = 5; f(y) = \frac{y+16}{4}, g(y) = \frac{y^2-4}{4}$
$\Rightarrow A = \int_c^d [f(y) - g(y)]\, dy = \int_{-4}^{5} \left(\frac{y+16}{4} - \frac{y^2-4}{4}\right) dy$
$= \frac{1}{4}\int_{-4}^{5} (y + 20 - y^2)\, dy = \frac{1}{4}\left[\frac{y^2}{2} + 20y - \frac{y^3}{3}\right]_{-4}^{5}$
$= \frac{1}{4}\left[\left(\frac{25}{2} + 100 - \frac{125}{3}\right) - \left(\frac{16}{2} - 80 + \frac{64}{3}\right)\right]$
$= \frac{1}{4}\left(\frac{9}{2} + 180 - 63\right) = \frac{1}{4}\left(\frac{9}{2} + 117\right) = \frac{1}{8}(9 + 234) = \frac{243}{8}$



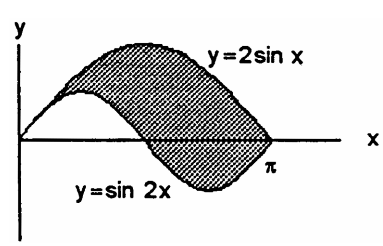
23. $f(x) = x, g(x) = \sin x, a = 0, b = \frac{\pi}{4}$
$\Rightarrow A = \int_a^b [f(x) - g(x)]\, dx = \int_0^{\pi/4} (x - \sin x)\, dx$
$= \left[\frac{x^2}{2} + \cos x\right]_0^{\pi/4} = \left(\frac{\pi^2}{32} + \frac{\sqrt{2}}{2}\right) - 1$



24. $f(x) = 1, g(x) = |\sin x|, a = -\frac{\pi}{2}, b = \frac{\pi}{2}$
$\Rightarrow A = \int_a^b [f(x) - g(x)]\, dx = \int_{-\pi/2}^{\pi/2} (1 - |\sin x|)\, dx$
$= \int_{-\pi/2}^{0} (1 + \sin x)\, dx + \int_0^{\pi/2} (1 - \sin x)\, dx$
$= 2\int_0^{\pi/2} (1 - \sin x)\, dx = 2[x + \cos x]_0^{\pi/2}$
$= 2\left(\frac{\pi}{2} - 1\right) = \pi - 2$



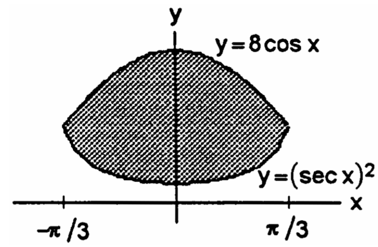
25. $a = 0, b = \pi, f(x) - g(x) = 2\sin x - \sin 2x$
$\Rightarrow A = \int_0^{\pi} (2\sin x - \sin 2x)\, dx = \left[-2\cos x + \frac{\cos 2x}{2}\right]_0^{\pi}$
$= \left[-2\cdot(-1) + \frac{1}{2}\right] - \left(-2\cdot 1 + \frac{1}{2}\right) = 4$



26. $a = -\frac{\pi}{3}$, $b = \frac{\pi}{3}$, $f(x) - g(x) = 8\cos x - \sec^2 x$

$\Rightarrow A = \int_{-\pi/3}^{\pi/3} (8\cos x - \sec^2 x)\,dx = [8\sin x - \tan x]_{-\pi/3}^{\pi/3}$

$= \left(8 \cdot \frac{\sqrt{3}}{2} - \sqrt{3}\right) - \left(-8 \cdot \frac{\sqrt{3}}{2} + \sqrt{3}\right) = 6\sqrt{3}$

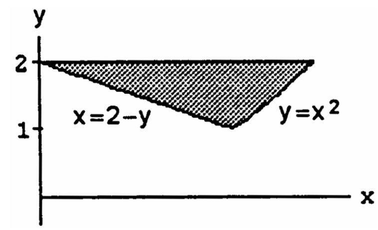


27. $f(y) = \sqrt{y}$, $g(y) = 2 - y$, $c = 1$, $d = 2$

$\Rightarrow A = \int_c^d [f(y) - g(y)]\,dy = \int_1^2 \left[\sqrt{y} - (2-y)\right] dy$

$= \int_1^2 \left(\sqrt{y} - 2 + y\right) dy = \left[\frac{2}{3} y^{3/2} - 2y + \frac{y^2}{2}\right]_1^2$

$= \left(\frac{4}{3}\sqrt{2} - 4 + 2\right) - \left(\frac{2}{3} - 2 + \frac{1}{2}\right) = \frac{4}{3}\sqrt{2} - \frac{7}{6} = \frac{8\sqrt{2}-7}{6}$

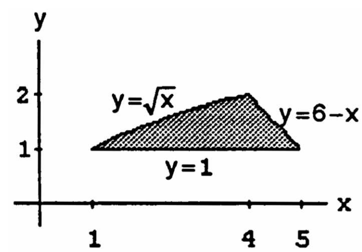


28. $f(y) = 6 - y$, $g(y) = y^2$, $c = 1$, $d = 2$

$\Rightarrow A = \int_c^d [f(y) - g(y)]\,dy = \int_1^2 (6 - y - y^2)\,dy$

$= \left[6y - \frac{y^2}{2} - \frac{y^3}{3}\right]_1^2 = \left(12 - 2 - \frac{8}{3}\right) - \left(6 - \frac{1}{2} - \frac{1}{3}\right)$

$= 4 - \frac{7}{3} + \frac{1}{2} = \frac{24 - 14 + 3}{6} = \frac{13}{6}$



29. $f(x) = x^3 - 3x^2 = x^2(x-3) \Rightarrow f'(x) = 3x^2 - 6x = 3x(x-2) \Rightarrow f' = +++\underset{0}{|}----\underset{2}{|}+++$

$\Rightarrow$ $f(0) = 0$ is a maximum and $f(2) = -4$ is a minimum. $A = -\int_0^3 (x^3 - 3x^2)\,dx = -\left[\frac{x^4}{4} - x^3\right]_0^3 = -\left(\frac{81}{4} - 27\right) = \frac{27}{4}$

30. $A = \int_0^a \left(a^{1/2} - x^{1/2}\right)^2 dx = \int_0^a \left(a - 2\sqrt{a}\,x^{1/2} + x\right) dx = \left[ax - \frac{4}{3}\sqrt{a}\,x^{3/2} + \frac{x^2}{2}\right]_0^a = a^2 - \frac{4}{3}\sqrt{a} \cdot a\sqrt{a} + \frac{a^2}{2}$

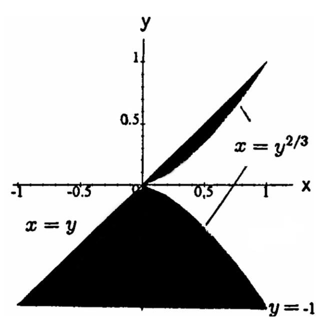
$= a^2\left(1 - \frac{4}{3} + \frac{1}{2}\right) = \frac{a^2}{6}(6 - 8 + 3) = \frac{a^2}{6}$

31. The area above the x-axis is $A_1 = \int_0^1 \left(y^{2/3} - y\right) dy$

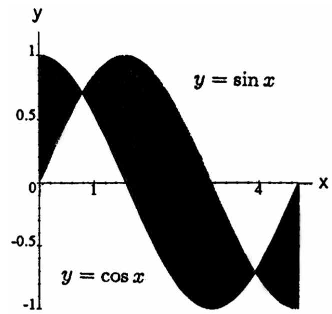
$= \left[\frac{3y^{5/3}}{5} - \frac{y^2}{2}\right]_0^1 = \frac{1}{10}$; the area below the x-axis is

$A_2 = \int_{-1}^0 \left(y^{2/3} - y\right) dy = \left[\frac{3y^{5/3}}{5} - \frac{y^2}{2}\right]_{-1}^0 = \frac{11}{10}$

$\Rightarrow$ the total area is $A_1 + A_2 = \frac{6}{5}$



32. $A = \int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\,dx$
$+ \int_{5\pi/4}^{3\pi/2} (\cos x - \sin x)\,dx = [\sin x + \cos x]_0^{\pi/4}$
$+ [-\cos x - \sin x]_{\pi/4}^{5\pi/4} + [\sin x + \cos x]_{5\pi/4}^{3\pi/2}$
$= \left[\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - (0+1)\right] + \left[\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)\right]$
$+ \left[(-1+0) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)\right] = \frac{8\sqrt{2}}{2} - 2 = 4\sqrt{2} - 2$



33. $y = x^2 + \int_1^x \frac{1}{t}\,dt \Rightarrow \frac{dy}{dx} = 2x + \frac{1}{x} \Rightarrow \frac{d^2y}{dx^2} = 2 - \frac{1}{x^2}$; $y(1) = 1 + \int_1^1 \frac{1}{t}\,dt = 1$ and $y'(1) = 2 + 1 = 3$

34. $y = \int_0^x \left(1 + 2\sqrt{\sec t}\right) dt \Rightarrow \frac{dy}{dx} = 1 + 2\sqrt{\sec x} \Rightarrow \frac{d^2y}{dx^2} = 2\left(\frac{1}{2}\right)(\sec x)^{-1/2}(\sec x \tan x) = \sqrt{\sec x}\,(\tan x)$;
$x = 0 \Rightarrow y = \int_0^0 \left(1 + 2\sqrt{\sec t}\right) dt = 0$ and $x = 0 \Rightarrow \frac{dy}{dx} = 1 + 2\sqrt{\sec 0} = 3$

35. $y = \int_5^x \frac{\sin t}{t}\,dt - 3 \Rightarrow \frac{dy}{dx} = \frac{\sin x}{x}$; $x = 5 \Rightarrow y = \int_5^5 \frac{\sin t}{t}\,dt - 3 = -3$

36. $y = \int_{-1}^x \sqrt{2 - \sin^2 t}\,dt + 2$ so that $\frac{dy}{dx} = \sqrt{2 - \sin^2 x}$; $x = -1 \Rightarrow y = \int_{-1}^{-1} \sqrt{2 - \sin^2 t}\,dt + 2 = 2$

37. Let $u = \cos x \Rightarrow du = -\sin x\,dx \Rightarrow -du = \sin x\,dx$
$\int 2(\cos x)^{-1/2} \sin x\,dx = \int 2u^{-1/2}(-du) = -2\int u^{-1/2}\,du = -2\left(\frac{u^{1/2}}{\frac{1}{2}}\right) + C = -4u^{1/2} + C = -4(\cos x)^{1/2} + C$

38. Let $u = \tan x \Rightarrow du = \sec^2 x\,dx$
$\int (\tan x)^{-3/2} \sec^2 x\,dx = \int u^{-3/2}\,du = \frac{u^{-1/2}}{\left(-\frac{1}{2}\right)} + C = -2u^{-1/2} + C = \frac{-2}{(\tan x)^{1/2}} + C$

39. Let $u = 2\theta + 1 \Rightarrow du = 2\,d\theta \Rightarrow \frac{1}{2}\,du = d\theta$
$\int [2\theta + 1 + 2\cos(2\theta + 1)]\,d\theta = \int (u + 2\cos u)\left(\frac{1}{2}\,du\right) = \frac{u^2}{4} + \sin u + C_1 = \frac{(2\theta+1)^2}{4} + \sin(2\theta + 1) + C_1$
$= \theta^2 + \theta + \sin(2\theta + 1) + C$, where $C = C_1 + \frac{1}{4}$ is still an arbitrary constant

40. Let $u = 2\theta - \pi \Rightarrow du = 2\,d\theta \Rightarrow \frac{1}{2}\,du = d\theta$
$\int \left(\frac{1}{\sqrt{2\theta - \pi}} + 2\sec^2(2\theta - \pi)\right) d\theta = \int \left(\frac{1}{\sqrt{u}} + 2\sec^2 u\right)\left(\frac{1}{2}\,du\right) = \frac{1}{2}\int \left(u^{-1/2} + 2\sec^2 u\right) du$
$= \frac{1}{2}\left(\frac{u^{1/2}}{\frac{1}{2}}\right) + \frac{1}{2}(2\tan u) + C = u^{1/2} + \tan u + C = (2\theta - \pi)^{1/2} + \tan(2\theta - \pi) + C$

41. $\int \left(t - \frac{2}{t}\right)\left(t + \frac{2}{t}\right) dt = \int \left(t^2 - \frac{4}{t^2}\right) dt = \int \left(t^2 - 4t^{-2}\right) dt = \frac{t^3}{3} - 4\left(\frac{t^{-1}}{-1}\right) + C = \frac{t^3}{3} + \frac{4}{t} + C$

42. $\int \frac{(t+1)^2 - 1}{t^4}\,dt = \int \frac{t^2 + 2t}{t^4}\,dt = \int \left(\frac{1}{t^2} + \frac{2}{t^3}\right) dt = \int \left(t^{-2} + 2t^{-3}\right) dt = \frac{t^{-1}}{(-1)} + 2\left(\frac{t^{-2}}{-2}\right) + C = -\frac{1}{t} - \frac{1}{t^2} + C$

43. Let $u = 2t^{3/2} \Rightarrow du = 3\sqrt{t}\,dt \Rightarrow \frac{1}{3}du = \sqrt{t}\,dt$
$\int \sqrt{t}\sin\left(2t^{3/2}\right)dt = \frac{1}{3}\int \sin u\,du = -\frac{1}{3}\cos u + C = -\frac{1}{3}\cos\left(2t^{3/2}\right) + C$

44. Let $u = 1 + \sec\theta \Rightarrow du = \sec\theta\tan\theta\, d\theta \Rightarrow \int \sec\theta\tan\theta\sqrt{1+\sec\theta}\, d\theta = \int u^{1/2} du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}(1+\sec\theta)^{3/2} + C$

45. $\int_{-1}^{1}(3x^2 - 4x + 7)\, dx = [x^3 - 2x^2 + 7x]_{-1}^{1} = [1^3 - 2(1)^2 + 7(1)] - [(-1)^3 - 2(-1)^2 + 7(-1)] = 6 - (-10) = 16$

46. $\int_{0}^{1}(8s^3 - 12s^2 + 5)\, ds = [2s^4 - 4s^3 + 5s]_{0}^{1} = [2(1)^4 - 4(1)^3 + 5(1)] - 0 = 3$

47. $\int_{1}^{2}\frac{4}{v^2}\, dv = \int_{1}^{2} 4v^{-2}\, dv = [-4v^{-1}]_{1}^{2} = \left(\frac{-4}{2}\right) - \left(\frac{-4}{1}\right) = 2$

48. $\int_{1}^{27} x^{-4/3}\, dx = \left[-3x^{-1/3}\right]_{1}^{27} = -3(27)^{-1/3} - \left(-3(1)^{-1/3}\right) = -3\left(\frac{1}{3}\right) + 3(1) = 2$

49. $\int_{1}^{4}\frac{dt}{t\sqrt{t}} = \int_{1}^{4}\frac{dt}{t^{3/2}} = \int_{1}^{4} t^{-3/2}\, dt = \left[-2t^{-1/2}\right]_{1}^{4} = \frac{-2}{\sqrt{4}} - \frac{(-2)}{\sqrt{1}} = 1$

50. Let $x = 1 + \sqrt{u} \Rightarrow dx = \frac{1}{2}u^{-1/2}\, du \Rightarrow 2\, dx = \frac{du}{\sqrt{u}}$; $u = 1 \Rightarrow x = 2, u = 4 \Rightarrow x = 3$

$\int_{1}^{4}\frac{\left(1+\sqrt{u}\right)^{1/2}}{\sqrt{u}}\, du = \int_{2}^{3} x^{1/2}(2\, dx) = \left[2\left(\frac{2}{3}\right)x^{3/2}\right]_{2}^{3} = \frac{4}{3}\left(3^{3/2}\right) - \frac{4}{3}\left(2^{3/2}\right) = 4\sqrt{3} - \frac{8}{3}\sqrt{2} = \frac{4}{3}\left(3\sqrt{3} - 2\sqrt{2}\right)$

51. Let $u = 2x + 1 \Rightarrow du = 2\, dx \Rightarrow 18\, du = 36\, dx$; $x = 0 \Rightarrow u = 1, x = 1 \Rightarrow u = 3$

$\int_{0}^{1}\frac{36\, dx}{(2x+1)^3} = \int_{1}^{3} 18u^{-3}\, du = \left[\frac{18u^{-2}}{-2}\right]_{1}^{3} = \left[\frac{-9}{u^2}\right]_{1}^{3} = \left(\frac{-9}{3^2}\right) - \left(\frac{-9}{1^2}\right) = 8$

52. Let $u = 7 - 5r \Rightarrow du = -5\, dr \Rightarrow -\frac{1}{5}\, du = dr$; $r = 0 \Rightarrow u = 7, r = 1 \Rightarrow u = 2$

$\int_{0}^{1}\frac{dr}{\sqrt[3]{(7-5r)^2}} = \int_{0}^{1}(7-5r)^{-2/3}\, dr = \int_{7}^{2} u^{-2/3}\left(-\frac{1}{5}\, du\right) = -\frac{1}{5}\left[3u^{1/3}\right]_{7}^{2} = \frac{3}{5}\left(\sqrt[3]{7} - \sqrt[3]{2}\right)$

53. Let $u = 1 - x^{2/3} \Rightarrow du = -\frac{2}{3}x^{-1/3}\, dx \Rightarrow -\frac{3}{2}\, du = x^{-1/3}\, dx$; $x = \frac{1}{8} \Rightarrow u = 1 - \left(\frac{1}{8}\right)^{2/3} = \frac{3}{4}$, $x = 1 \Rightarrow u = 1 - 1^{2/3} = 0$

$\int_{1/8}^{1} x^{-1/3}\left(1 - x^{2/3}\right)^{3/2} dx = \int_{3/4}^{0} u^{3/2}\left(-\frac{3}{2}\, du\right) = \left[\left(-\frac{3}{2}\right)\left(\frac{u^{5/2}}{\frac{5}{2}}\right)\right]_{3/4}^{0} = \left[-\frac{3}{5}u^{5/2}\right]_{3/4}^{0} = -\frac{3}{5}(0)^{5/2} - \left(-\frac{3}{5}\right)\left(\frac{3}{4}\right)^{5/2}$

$= \frac{27\sqrt{3}}{160}$

54. Let $u = 1 + 9x^4 \Rightarrow du = 36x^3\, dx \Rightarrow \frac{1}{36}\, du = x^3\, dx$; $x = 0 \Rightarrow u = 1, x = \frac{1}{2} \Rightarrow u = 1 + 9\left(\frac{1}{2}\right)^4 = \frac{25}{16}$

$\int_{0}^{1/2} x^3\left(1 + 9x^4\right)^{-3/2} dx = \int_{1}^{25/16} u^{-3/2}\left(\frac{1}{36}\, du\right) = \left[\frac{1}{36}\left(\frac{u^{-1/2}}{-\frac{1}{2}}\right)\right]_{1}^{25/16} = \left[-\frac{1}{18}u^{-1/2}\right]_{1}^{25/16}$

$= -\frac{1}{18}\left(\frac{25}{16}\right)^{-1/2} - \left(-\frac{1}{18}(1)^{-1/2}\right) = \frac{1}{90}$

55. Let $u = 5r \Rightarrow du = 5\, dr \Rightarrow \frac{1}{5}\, du = dr$; $r = 0 \Rightarrow u = 0, r = \pi \Rightarrow u = 5\pi$

$\int_{0}^{\pi}\sin^2 5r\, dr = \int_{0}^{5\pi}(\sin^2 u)\left(\frac{1}{5}\, du\right) = \frac{1}{5}\left[\frac{u}{2} - \frac{\sin 2u}{4}\right]_{0}^{5\pi} = \left(\frac{\pi}{2} - \frac{\sin 10\pi}{20}\right) - \left(0 - \frac{\sin 0}{20}\right) = \frac{\pi}{2}$

56. Let $u = 4t - \frac{\pi}{4} \Rightarrow du = 4\, dt \Rightarrow \frac{1}{4}\, du = dt$; $t = 0 \Rightarrow u = -\frac{\pi}{4}$, $t = \frac{\pi}{4} \Rightarrow u = \frac{3\pi}{4}$

$\int_{0}^{\pi/4}\cos^2\left(4t - \frac{\pi}{4}\right) dt = \int_{-\pi/4}^{3\pi/4}(\cos^2 u)\left(\frac{1}{4}\, du\right) = \frac{1}{4}\left[\frac{u}{2} + \frac{\sin 2u}{4}\right]_{-\pi/4}^{3\pi/4} = \frac{1}{4}\left(\frac{3\pi}{8} + \frac{\sin\left(\frac{3\pi}{2}\right)}{4}\right) - \frac{1}{4}\left(-\frac{\pi}{8} + \frac{\sin\left(-\frac{\pi}{2}\right)}{4}\right)$

$= \frac{\pi}{8} - \frac{1}{16} + \frac{1}{16} = \frac{\pi}{8}$

57. $\int_0^{\pi/3} \sec^2 \theta \, d\theta = [\tan \theta]_0^{\pi/3} = \tan \frac{\pi}{3} - \tan 0 = \sqrt{3}$

58. $\int_{\pi/4}^{3\pi/4} \csc^2 x \, dx = [-\cot x]_{\pi/4}^{3\pi/4} = \left(-\cot \frac{3\pi}{4}\right) - \left(-\cot \frac{\pi}{4}\right) = 2$

59. Let $u = \frac{x}{6} \Rightarrow du = \frac{1}{6} dx \Rightarrow 6\,du = dx$; $x = \pi \Rightarrow u = \frac{\pi}{6}$, $x = 3\pi \Rightarrow u = \frac{\pi}{2}$
$\int_{\pi}^{3\pi} \cot^2 \frac{x}{6} \, dx = \int_{\pi/6}^{\pi/2} 6 \cot^2 u \, du = 6\int_{\pi/6}^{\pi/2} (\csc^2 u - 1) \, du = [6(-\cot u - u)]_{\pi/6}^{\pi/2} = 6\left(-\cot \frac{\pi}{2} - \frac{\pi}{2}\right) - 6\left(-\cot \frac{\pi}{6} - \frac{\pi}{6}\right)$
$= 6\sqrt{3} - 2\pi$

60. Let $u = \frac{\theta}{3} \Rightarrow du = \frac{1}{3} d\theta \Rightarrow 3\,du = d\theta$; $\theta = 0 \Rightarrow u = 0$, $\theta = \pi \Rightarrow u = \frac{\pi}{3}$
$\int_0^{\pi} \tan^2 \frac{\theta}{3} \, d\theta = \int_0^{\pi} \left(\sec^2 \frac{\theta}{3} - 1\right) d\theta = \int_0^{\pi/3} 3\left(\sec^2 u - 1\right) du = [3 \tan u - 3u]_0^{\pi/3} = \left[3 \tan \frac{\pi}{3} - 3\left(\frac{\pi}{3}\right)\right] - (3 \tan 0 - 0)$
$= 3\sqrt{3} - \pi$

61. $\int_{-\pi/3}^{0} \sec x \tan x \, dx = [\sec x]_{-\pi/3}^{0} = \sec 0 - \sec\left(-\frac{\pi}{3}\right) = 1 - 2 = -1$

62. $\int_{\pi/4}^{3\pi/4} \csc z \cot z \, dz = [-\csc z]_{\pi/4}^{3\pi/4} = \left(-\csc \frac{3\pi}{4}\right) - \left(-\csc \frac{\pi}{4}\right) = -\sqrt{2} + \sqrt{2} = 0$

63. Let $u = \sin x \Rightarrow du = \cos x \, dx$; $x = 0 \Rightarrow u = 0$, $x = \frac{\pi}{2} \Rightarrow u = 1$
$\int_0^{\pi/2} 5(\sin x)^{3/2} \cos x \, dx = \int_0^1 5u^{3/2} \, du = \left[5\left(\frac{2}{5}\right) u^{5/2}\right]_0^1 = \left[2u^{5/2}\right]_0^1 = 2(1)^{5/2} - 2(0)^{5/2} = 2$

64. Let $u = 1 - x^2 \Rightarrow du = -2x \, dx \Rightarrow -du = 2x \, dx$; $x = -1 \Rightarrow u = 0$, $x = 1 \Rightarrow u = 0$
$\int_{-1}^{1} 2x \sin\left(1 - x^2\right) dx = \int_0^0 -\sin u \, du = 0$

65. Let $u = \sin 3x \Rightarrow du = 3 \cos 3x \, dx \Rightarrow \frac{1}{3} du = \cos 3x \, dx$; $x = -\frac{\pi}{2} \Rightarrow u = \sin\left(-\frac{3\pi}{2}\right) = 1$, $x = \frac{\pi}{2} \Rightarrow u = \sin\left(\frac{3\pi}{2}\right) = -1$
$\int_{-\pi/2}^{\pi/2} 15 \sin^4 3x \cos 3x \, dx = \int_1^{-1} 15u^4 \left(\frac{1}{3} du\right) = \int_1^{-1} 5u^4 \, du = \left[u^5\right]_1^{-1} = (-1)^5 - (1)^5 = -2$

66. Let $u = \cos\left(\frac{x}{2}\right) \Rightarrow du = -\frac{1}{2} \sin\left(\frac{x}{2}\right) dx \Rightarrow -2\,du = \sin\left(\frac{x}{2}\right) dx$; $x = 0 \Rightarrow u = \cos\left(\frac{0}{2}\right) = 1$, $x = \frac{2\pi}{3} \Rightarrow u = \cos\left(\frac{\frac{2\pi}{3}}{2}\right) = \frac{1}{2}$
$\int_0^{2\pi/3} \cos^{-4}\left(\frac{x}{2}\right) \sin\left(\frac{x}{2}\right) dx = \int_1^{1/2} u^{-4}(-2\,du) = \left[-2\left(\frac{u^{-3}}{-3}\right)\right]_1^{1/2} = \frac{2}{3}\left(\frac{1}{2}\right)^{-3} - \frac{2}{3}(1)^{-3} = \frac{2}{3}(8 - 1) = \frac{14}{3}$

67. Let $u = 1 + 3\sin^2 x \Rightarrow du = 6 \sin x \cos x \, dx \Rightarrow \frac{1}{2} du = 3 \sin x \cos x \, dx$; $x = 0 \Rightarrow u = 1$, $x = \frac{\pi}{2} \Rightarrow u = 1 + 3\sin^2 \frac{\pi}{2} = 4$
$\int_0^{\pi/2} \frac{3 \sin x \cos x}{\sqrt{1 + 3\sin^2 x}} \, dx = \int_1^4 \frac{1}{\sqrt{u}} \left(\frac{1}{2} du\right) = \int_1^4 \frac{1}{2} u^{-1/2} \, du = \left[\frac{1}{2}\left(\frac{u^{1/2}}{\frac{1}{2}}\right)\right]_1^4 = \left[u^{1/2}\right]_1^4 = 4^{1/2} - 1^{1/2} = 1$

68. Let $u = 1 + 7\tan x \Rightarrow du = 7\sec^2 x \, dx \Rightarrow \frac{1}{7} du = \sec^2 x \, dx$; $x = 0 \Rightarrow u = 1 + 7 \tan 0 = 1$, $x = \frac{\pi}{4}$
$\Rightarrow u = 1 + 7 \tan \frac{\pi}{4} = 8$
$\int_0^{\pi/4} \frac{\sec^2 x}{(1 + 7\tan x)^{2/3}} \, dx = \int_1^8 \frac{1}{u^{2/3}} \left(\frac{1}{7} du\right) = \int_1^8 \frac{1}{7} u^{-2/3} \, du = \left[\frac{1}{7}\left(\frac{u^{1/3}}{\frac{1}{3}}\right)\right]_1^8 = \left[\frac{3}{7} u^{1/3}\right]_1^8 = \frac{3}{7}(8)^{1/3} - \frac{3}{7}(1)^{1/3} = \frac{3}{7}$

69. Let $u = \sec\theta \Rightarrow du = \sec\theta\tan\theta\, d\theta;\ \theta = 0 \Rightarrow u = \sec 0 = 1,\ \theta = \frac{\pi}{3} \Rightarrow u = \sec\frac{\pi}{3} = 2$

$$\int_0^{\pi/3} \frac{\tan\theta}{\sqrt{2\sec\theta}}\, d\theta = \int_0^{\pi/3} \frac{\sec\theta\tan\theta}{\sec\theta\sqrt{2\sec\theta}}\, d\theta = \int_0^{\pi/3} \frac{\sec\theta\tan\theta}{\sqrt{2}\,(\sec\theta)^{3/2}}\, d\theta = \int_1^2 \frac{1}{\sqrt{2}\,u^{3/2}}\, du = \frac{1}{\sqrt{2}} \int_1^2 u^{-3/2}\, du$$
$$= \frac{1}{\sqrt{2}}\left[\frac{u^{-1/2}}{\left(-\frac{1}{2}\right)}\right]_1^2 = \left[-\frac{2}{\sqrt{2u}}\right]_1^2 = -\frac{2}{\sqrt{2(2)}} - \left(-\frac{2}{\sqrt{2(1)}}\right) = \sqrt{2} - 1$$

70. Let $u = \sin\sqrt{t} \Rightarrow du = \left(\cos\sqrt{t}\right)\left(\frac{1}{2}t^{-1/2}\right) dt = \frac{\cos\sqrt{t}}{2\sqrt{t}}\, dt \Rightarrow 2\, du = \frac{\cos\sqrt{t}}{\sqrt{t}}\, dt;\ t = \frac{\pi^2}{36} \Rightarrow u = \sin\frac{\pi}{6} = \frac{1}{2}$,
$t = \frac{\pi^2}{4} \Rightarrow u = \sin\frac{\pi}{2} = 1$

$$\int_{\pi^2/36}^{\pi^2/4} \frac{\cos\sqrt{t}}{\sqrt{t\sin\sqrt{t}}}\, dt = \int_{1/2}^{1} \frac{1}{\sqrt{u}}\,(2\, du) = 2\int_{1/2}^{1} u^{-1/2}\, du = \left[4\sqrt{u}\right]_{1/2}^{1} = 4\sqrt{1} - 4\sqrt{\tfrac{1}{2}} = 2\left(2 - \sqrt{2}\right)$$

71. (a) $\operatorname{av}(f) = \frac{1}{1-(-1)} \int_{-1}^{1} (mx + b)\, dx = \frac{1}{2}\left[\frac{mx^2}{2} + bx\right]_{-1}^{1} = \frac{1}{2}\left[\left(\frac{m(1)^2}{2} + b(1)\right) - \left(\frac{m(-1)^2}{2} + b(-1)\right)\right] = \frac{1}{2}(2b) = b$

(b) $\operatorname{av}(f) = \frac{1}{k-(-k)} \int_{-k}^{k} (mx + b)\, dx = \frac{1}{2k}\left[\frac{mx^2}{2} + bx\right]_{-k}^{k} = \frac{1}{2k}\left[\left(\frac{m(k)^2}{2} + b(k)\right) - \left(\frac{m(-k)^2}{2} + b(-k)\right)\right] = \frac{1}{2k}(2bk) = b$

72. (a) $y_{av} = \frac{1}{3-0}\int_0^3 \sqrt{3x}\, dx = \frac{1}{3}\int_0^3 \sqrt{3}\, x^{1/2}\, dx = \frac{\sqrt{3}}{3}\left[\frac{2}{3}x^{3/2}\right]_0^3 = \frac{\sqrt{3}}{3}\left[\frac{2}{3}(3)^{3/2} - \frac{2}{3}(0)^{3/2}\right] = \frac{\sqrt{3}}{3}\left(2\sqrt{3}\right) = 2$

(b) $y_{av} = \frac{1}{a-0}\int_0^a \sqrt{ax}\, dx = \frac{1}{a}\int_0^a \sqrt{a}\, x^{1/2}\, dx = \frac{\sqrt{a}}{a}\left[\frac{2}{3}x^{3/2}\right]_0^a = \frac{\sqrt{a}}{a}\left(\frac{2}{3}(a)^{3/2} - \frac{2}{3}(0)^{3/2}\right) = \frac{\sqrt{a}}{a}\left(\frac{2}{3}a\sqrt{a}\right) = \frac{2}{3}a$

73. $f'_{av} = \frac{1}{b-a}\int_a^b f'(x)\, dx = \frac{1}{b-a}\left[f(x)\right]_a^b = \frac{1}{b-a}\left[f(b) - f(a)\right] = \frac{f(b)-f(a)}{b-a}$ so the average value of f' over $[a, b]$ is the slope of the secant line joining the points $(a, f(a))$ and $(b, f(b))$, which is the average rate of change of f over $[a, b]$.

74. Yes, because the average value of f on $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)\, dx$. If the length of the interval is 2, then $b - a = 2$ and the average value of the function is $\frac{1}{2}\int_a^b f(x)\, dx$.

75. We want to evaluate

$$\frac{1}{365-0}\int_0^{365} f(x)\, dx = \frac{1}{365}\int_0^{365}\left(37\sin\left[\frac{2\pi}{365}(x-101)\right] + 25\right)dx = \frac{37}{365}\int_0^{365}\sin\left[\frac{2\pi}{365}(x-101)\right]dx + \frac{25}{365}\int_0^{365} dx$$

Notice that the period of $y = \sin\left[\frac{2\pi}{365}(x-101)\right]$ is $\frac{2\pi}{\frac{2\pi}{365}} = 365$ and that we are integrating this function over an iterval of length 365. Thus the value of $\frac{37}{365}\int_0^{365}\sin\left[\frac{2\pi}{365}(x-101)\right]dx + \frac{25}{365}\int_0^{365} dx$ is $\frac{37}{365}\cdot 0 + \frac{25}{365}\cdot 365 = 25$.

76. $$\frac{1}{675-20}\int_{20}^{675}\left(8.27 + 10^{-5}(26T - 1.87T^2)\right)dT = \frac{1}{655}\left[8.27T + \frac{26T^2}{2\cdot 10^5} - \frac{1.87T^3}{3\cdot 10^5}\right]_{20}^{675}$$
$$= \frac{1}{655}\left(\left[8.27(675) + \frac{26(675)^2}{2\cdot 10^5} - \frac{1.87(675)^3}{3\cdot 10^5}\right] - \left[8.27(20) + \frac{26(20)^2}{2\cdot 10^5} - \frac{1.87(20)^3}{3\cdot 10^5}\right]\right) \approx \frac{1}{655}(3724.44 - 165.40)$$

$= 5.43 =$ the average value of C_v on $[20, 675]$. To find the temperature T at which $C_v = 5.43$, solve $5.43 = 8.27 + 10^{-5}(26T - 1.87T^2)$ for T. We obtain $1.87T^2 - 26T - 284000 = 0$
$\Rightarrow T = \frac{26 \pm \sqrt{(26)^2 - 4(1.87)(-284000)}}{2(1.87)} = \frac{26 \pm \sqrt{2124996}}{3.74}$. So $T = -382.82$ or $T = 396.72$. Only $T = 396.72$ lies in the interval $[20, 675]$, so $T = 396.72°$C.

77. $\frac{dy}{dx} = \sqrt{2 + \cos^3 x}$

78. $\frac{dy}{dx} = \sqrt{2 + \cos^3(7x^2)} \cdot \frac{d}{dx}(7x^2) = 14x\sqrt{2 + \cos^3(7x^2)}$

79. $\frac{dy}{dx} = \frac{d}{dx}\left(-\int_1^x \frac{6}{3+t^4} dt\right) = -\frac{6}{3+x^4}$

80. $\frac{dy}{dx} = \frac{d}{dx}\left(\int_{\sec x}^2 \frac{1}{t^2+1} dt\right) = -\frac{d}{dx}\left(\int_2^{\sec x} \frac{1}{t^2+1} dt\right) = -\frac{1}{\sec^2 x+1}\frac{d}{dx}(\sec x) = -\frac{\sec x \tan x}{1+\sec^2 x}$

81. Yes. The function f, being differentiable on [a, b], is then continuous on [a, b]. The Fundamental Theorem of Calculus says that every continuous function on [a, b] is the derivative of a function on [a, b].

82. The second part of the Fundamental Theorem of Calculus states that if F(x) is an antiderivative of f(x) on [a, b], then $\int_a^b f(x)\,dx = F(b) - F(a)$. In particular, if F(x) is an antiderivaitve of $\sqrt{1+x^4}$ on [0, 1], then $\int_0^1 \sqrt{1+x^4}\,dx = F(1) - F(0)$.

83. $y = \int_x^1 \sqrt{1+t^2}\,dt = -\int_1^x \sqrt{1+t^2}\,dt \Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left[-\int_1^x \sqrt{1+t^2}\,dt\right] = -\frac{d}{dx}\left[\int_1^x \sqrt{1+t^2}\,dt\right] = -\sqrt{1+x^2}$

84. $y = \int_{\cos x}^0 \frac{1}{1-t^2}\,dt = -\int_0^{\cos x} \frac{1}{1-t^2}\,dt \Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left[-\int_0^{\cos x} \frac{1}{1-t^2}\,dt\right] = -\frac{d}{dx}\left[\int_0^{\cos x} \frac{1}{1-t^2}\,dt\right]$
$= -\left(\frac{1}{1-\cos^2 x}\right)\left(\frac{d}{dx}(\cos x)\right) = -\left(\frac{1}{\sin^2 x}\right)(-\sin x) = \frac{1}{\sin x} = \csc x$

85. We estimate the area A using midpoints of the vertical intervals, and we will estimate the width of the parking lot on each interval by averaging the widths at top and bottom. This gives the estimate
$A \approx 15 \cdot \left(\frac{0+36}{2} + \frac{36+54}{2} + \frac{54+51}{2} + \frac{51+49.5}{2} + \frac{49.5+54}{2} + \frac{54+64.4}{2} + \frac{64.4+67.5}{2} + \frac{67.5+42}{2}\right)$
$A \approx 5961\ \text{ft}^2$. The cost is Area $\cdot$ (\$2.10/ft^2) $\approx (5961\ \text{ft}^2)$ (\$2.10/ft^2) = \$12,518.10 $\Rightarrow$ the job cannot be done for \$11,000.

86. (a) Before the chute opens for A, $a = -32\ \text{ft/sec}^2$. Since the helicopter is hovering, $v_0 = 0$ ft/sec $\Rightarrow v = \int -32\,dt = -32t + v_0 = -32t$. Then $s_0 = 6400$ ft $\Rightarrow s = \int -32t\,dt = -16t^2 + s_0 = -16t^2 + 6400$. At $t = 4$ sec, $s = -16(4)^2 + 6400 = 6144$ ft when A's chute opens;
(b) For B, $s_0 = 7000$ ft, $v_0 = 0$, $a = -32\ \text{ft/sec}^2 \Rightarrow v = \int -32\,dt = -32t + v_0 = -32t \Rightarrow s = \int -32t\,dt = -16t^2 + s_0 = -16t^2 + 7000$. At $t = 13$ sec, $s = -16(13)^2 + 7000 = 4296$ ft when B's chute opens;
(c) After the chutes open, $v = -16$ ft/sec $\Rightarrow s = \int -16\,dt = -16t + s_0$. For A, $s_0 = 6144$ ft and for B, $s_0 = 4296$ ft. Therefore, for A, $s = -16t + 6144$ and for B, $s = -16t + 4296$. When they hit the ground, $s = 0 \Rightarrow$ for A, $0 = -16t + 6144 \Rightarrow t = \frac{6144}{16} = 384$ seconds, and for B, $0 = -16t + 4296 \Rightarrow t = \frac{4296}{16} = 268.5$ seconds to hit the ground after the chutes open. Since B's chute opens 58 seconds after A's opens $\Rightarrow$ B hits the ground first.

CHAPTER 5 ADDITIONAL AND ADVANCED EXERCISES

1. (a) Yes, because $\int_0^1 f(x)\,dx = \frac{1}{7}\int_0^1 7f(x)\,dx = \frac{1}{7}(7) = 1$
(b) No. For example, $\int_0^1 8x\,dx = \left[4x^2\right]_0^1 = 4$, but $\int_0^1 \sqrt{8x}\,dx = \left[2\sqrt{2}\left(\frac{x^{3/2}}{\frac{3}{2}}\right)\right]_0^1 = \frac{4\sqrt{2}}{3}\left(1^{3/2} - 0^{3/2}\right) = \frac{4\sqrt{2}}{3} \neq \sqrt{4}$

2. (a) True: $\int_5^2 f(x)\,dx = -\int_2^5 f(x)\,dx = -3$
(b) True: $\int_{-2}^5 [f(x)+g(x)]\,dx = \int_{-2}^5 f(x)\,dx + \int_{-2}^5 g(x)\,dx = \int_{-2}^2 f(x)\,dx + \int_2^5 f(x)\,dx + \int_{-2}^5 g(x)\,dx = 4 + 3 + 2 = 9$

(c) False: $\int_{-2}^{5} f(x)\,dx = 4 + 3 = 7 > 2 = \int_{-2}^{5} g(x)\,dx \Rightarrow \int_{-2}^{5} [f(x) - g(x)]\,dx > 0 \Rightarrow \int_{-2}^{5} [g(x) - f(x)]\,dx < 0.$

On the other hand, $f(x) \le g(x) \Rightarrow [g(x) - f(x)] \ge 0 \Rightarrow \int_{-2}^{5} [g(x) - f(x)]\,dx \ge 0$ which is a contradiction.

3. $y = \frac{1}{a}\int_0^x f(t)\sin a(x-t)\,dt = \frac{1}{a}\int_0^x f(t)\sin ax\cos at\,dt - \frac{1}{a}\int_0^x f(t)\cos ax\sin at\,dt$

$= \frac{\sin ax}{a}\int_0^x f(t)\cos at\,dt - \frac{\cos ax}{a}\int_0^x f(t)\sin at\,dt \Rightarrow \frac{dy}{dx} = \cos ax\left(\int_0^x f(t)\cos at\,dt\right)$

$+ \frac{\sin ax}{a}\left(\frac{d}{dx}\int_0^x f(t)\cos at\,dt\right) + \sin ax\int_0^x f(t)\sin at\,dt - \frac{\cos ax}{a}\left(\frac{d}{dx}\int_0^x f(t)\sin at\,dt\right)$

$= \cos ax\int_0^x f(t)\cos at\,dt + \frac{\sin ax}{a}(f(x)\cos ax) + \sin ax\int_0^x f(t)\sin at\,dt - \frac{\cos ax}{a}(f(x)\sin ax)$

$\Rightarrow \frac{dy}{dx} = \cos ax\int_0^x f(t)\cos at\,dt + \sin ax\int_0^x f(t)\sin at\,dt$. Next,

$\frac{d^2y}{dx^2} = -a\sin ax\int_0^x f(t)\cos at\,dt + (\cos ax)\left(\frac{d}{dx}\int_0^x f(t)\cos at\,dt\right) + a\cos ax\int_0^x f(t)\sin at\,dt$

$+ (\sin ax)\left(\frac{d}{dx}\int_0^x f(t)\sin at\,dt\right) = -a\sin ax\int_0^x f(t)\cos at\,dt + (\cos ax)f(x)\cos ax$

$+ a\cos ax\int_0^x f(t)\sin at\,dt + (\sin ax)f(x)\sin ax = -a\sin ax\int_0^x f(t)\cos at\,dt + a\cos ax\int_0^x f(t)\sin at\,dt + f(x).$

Therefore, $y'' + a^2y = a\cos ax\int_0^x f(t)\sin at\,dt - a\sin ax\int_0^x f(t)\cos at\,dt + f(x)$

$+ a^2\left(\frac{\sin ax}{a}\int_0^x f(t)\cos at\,dt - \frac{\cos ax}{a}\int_0^x f(t)\sin at\,dt\right) = f(x)$. Note also that $y'(0) = y(0) = 0$.

4. $x = \int_0^y \frac{1}{\sqrt{1+4t^2}}\,dt \Rightarrow \frac{d}{dx}(x) = \frac{d}{dx}\int_0^y \frac{1}{\sqrt{1+4t^2}}\,dt = \frac{d}{dy}\left[\int_0^y \frac{1}{\sqrt{1+4t^2}}\,dt\right]\left(\frac{dy}{dx}\right)$ from the chain rule

$\Rightarrow 1 = \frac{1}{\sqrt{1+4y^2}}\left(\frac{dy}{dx}\right) \Rightarrow \frac{dy}{dx} = \sqrt{1+4y^2}$. Then $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\sqrt{1+4y^2}\right) = \frac{d}{dy}\left(\sqrt{1+4y^2}\right)\left(\frac{dy}{dx}\right)$

$= \frac{1}{2}\left(1+4y^2\right)^{-1/2}(8y)\left(\frac{dy}{dx}\right) = \frac{4y\left(\frac{dy}{dx}\right)}{\sqrt{1+4y^2}} = \frac{4y\left(\sqrt{1+4y^2}\right)}{\sqrt{1+4y^2}} = 4y$. Thus $\frac{d^2y}{dx^2} = 4y$, and the constant of proportionality is 4.

5. (a) $\int_0^{x^2} f(t)\,dt = x\cos\pi x \Rightarrow \frac{d}{dx}\int_0^{x^2} f(t)\,dt = \cos\pi x - \pi x\sin\pi x \Rightarrow f\left(x^2\right)(2x) = \cos\pi x - \pi x\sin\pi x$

$\Rightarrow f\left(x^2\right) = \frac{\cos\pi x - \pi x\sin\pi x}{2x}$. Thus, $x = 2 \Rightarrow f(4) = \frac{\cos 2\pi - 2\pi\sin 2\pi}{4} = \frac{1}{4}$

(b) $\int_0^{f(x)} t^2\,dt = \left[\frac{t^3}{3}\right]_0^{f(x)} = \frac{1}{3}(f(x))^3 \Rightarrow \frac{1}{3}(f(x))^3 = x\cos\pi x \Rightarrow (f(x))^3 = 3x\cos\pi x \Rightarrow f(x) = \sqrt[3]{3x\cos\pi x}$

$\Rightarrow f(4) = \sqrt[3]{3(4)\cos 4\pi} = \sqrt[3]{12}$

6. $\int_0^a f(x)\,dx = \frac{a^2}{2} + \frac{a}{2}\sin a + \frac{\pi}{2}\cos a$. Let $F(a) = \int_0^a f(t)\,dt \Rightarrow f(a) = F'(a)$. Now $F(a) = \frac{a^2}{2} + \frac{a}{2}\sin a + \frac{\pi}{2}\cos a$

$\Rightarrow f(a) = F'(a) = a + \frac{1}{2}\sin a + \frac{a}{2}\cos a - \frac{\pi}{2}\sin a \Rightarrow f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + \frac{1}{2}\sin\frac{\pi}{2} + \frac{\left(\frac{\pi}{2}\right)}{2}\cos\frac{\pi}{2} - \frac{\pi}{2}\sin\frac{\pi}{2} = \frac{\pi}{2} + \frac{1}{2} - \frac{\pi}{2} = \frac{1}{2}$

7. $\int_1^b f(x)\,dx = \sqrt{b^2+1} - \sqrt{2} \Rightarrow f(b) = \frac{d}{db}\int_1^b f(x)\,dx = \frac{1}{2}\left(b^2+1\right)^{-1/2}(2b) = \frac{b}{\sqrt{b^2+1}} \Rightarrow f(x) = \frac{x}{\sqrt{x^2+1}}$

8. The derivative of the left side of the equation is: $\frac{d}{dx}\left[\int_0^x\left[\int_0^u f(t)\,dt\right]du\right] = \int_0^x f(t)\,dt$; the derivative of the right side of the equation is: $\frac{d}{dx}\left[\int_0^x f(u)(x-u)\,du\right] = \frac{d}{dx}\int_0^x f(u)\,x\,du - \frac{d}{dx}\int_0^x u\,f(u)\,du$

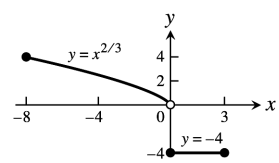
$= \frac{d}{dx}\left[x\int_0^x f(u)\,du\right] - \frac{d}{dx}\int_0^x u\,f(u)\,du = \int_0^x f(u)\,du + x\left[\frac{d}{dx}\int_0^x f(u)\,du\right] - xf(x) = \int_0^x f(u)\,du + xf(x) - xf(x)$

$= \int_0^x f(u)\,du$. Since each side has the same derivative, they differ by a constant, and since both sides equal 0 when $x = 0$, the constant must be 0. Therefore, $\int_0^x \left[\int_0^u f(t)\,dt\right] du = \int_0^x f(u)(x-u)\,du$.

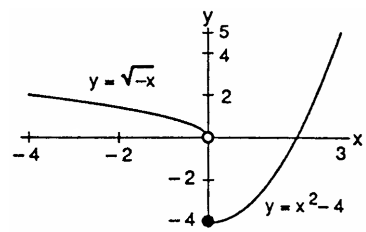
9. $\frac{dy}{dx} = 3x^2 + 2 \Rightarrow y = \int (3x^2+2)\,dx = x^3 + 2x + C$. Then $(1, -1)$ on the curve $\Rightarrow 1^3 + 2(1) + C = -1 \Rightarrow C = -4$ $\Rightarrow y = x^3 + 2x - 4$

10. The acceleration due to gravity downward is -32 ft/sec^2 $\Rightarrow v = \int -32\,dt = -32t + v_0$, where v_0 is the initial velocity $\Rightarrow v = -32t + 32 \Rightarrow s = \int (-32t + 32)\,dt = -16t^2 + 32t + C$. If the release point, at $t = 0$, is $s = 0$, then $C = 0 \Rightarrow s = -16t^2 + 32t$. Then $s = 17 \Rightarrow 17 = -16t^2 + 32t \Rightarrow 16t^2 - 32t + 17 = 0$. The discriminant of this quadratic equation is -64 which says there is no real time when $s = 17$ ft. You had better duck.

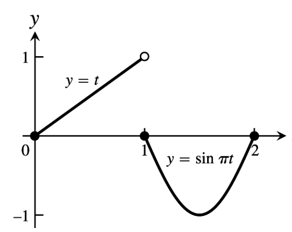
11. $\int_{-8}^{3} f(x)\,dx = \int_{-8}^{0} x^{2/3}\,dx + \int_0^3 -4\,dx$
$= \left[\frac{3}{5}x^{5/3}\right]_{-8}^{0} + [-4x]_0^3$
$= \left(0 - \frac{3}{5}(-8)^{5/3}\right) + (-4(3) - 0) = \frac{96}{5} - 12$
$= \frac{36}{5}$



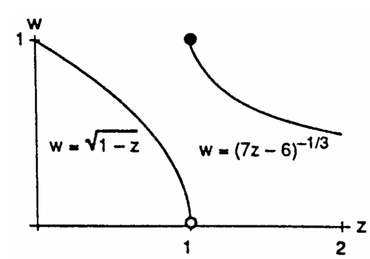
12. $\int_{-4}^{3} f(x)\,dx = \int_{-4}^{0} \sqrt{-x}\,dx + \int_0^3 (x^2 - 4)\,dx$
$= \left[-\frac{2}{3}(-x)^{3/2}\right]_{-4}^{0} + \left[\frac{x^3}{3} - 4x\right]_0^3$
$= \left[0 - \left(-\frac{2}{3}(4)^{3/2}\right)\right] + \left[\left(\frac{3^3}{3} - 4(3)\right) - 0\right]$
$= \frac{16}{3} - 3 = \frac{7}{3}$



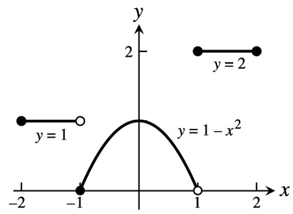
13. $\int_0^2 g(t)\,dt = \int_0^1 t\,dt + \int_1^2 \sin \pi t\,dt$
$= \left[\frac{t^2}{2}\right]_0^1 + \left[-\frac{1}{\pi}\cos \pi t\right]_1^2$
$= \left(\frac{1}{2} - 0\right) + \left[-\frac{1}{\pi}\cos 2\pi - \left(-\frac{1}{\pi}\cos \pi\right)\right]$
$= \frac{1}{2} - \frac{2}{\pi}$



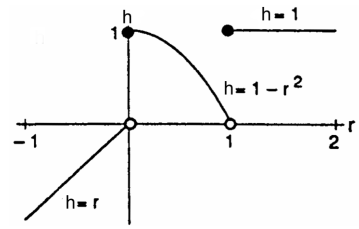
14. $\int_0^2 h(z)\,dz = \int_0^1 \sqrt{1-z}\,dz + \int_1^2 (7z-6)^{-1/3}\,dz$
$= \left[-\frac{2}{3}(1-z)^{3/2}\right]_0^1 + \left[\frac{3}{14}(7z-6)^{2/3}\right]_1^2$
$= \left[-\frac{2}{3}(1-1)^{3/2} - \left(-\frac{2}{3}(1-0)^{3/2}\right)\right]$
$+ \left[\frac{3}{14}(7(2)-6)^{2/3} - \frac{3}{14}(7(1)-6)^{2/3}\right]$
$= \frac{2}{3} + \left(\frac{6}{7} - \frac{3}{14}\right) = \frac{55}{42}$



15. $\int_{-2}^{2} f(x)\,dx = \int_{-2}^{-1} dx + \int_{-1}^{1} (1-x^2)\,dx + \int_{1}^{2} 2\,dx$
$= [x]_{-2}^{-1} + \left[x - \frac{x^3}{3}\right]_{-1}^{1} + [2x]_1^2$
$= (-1-(-2)) + \left[\left(1-\frac{1^3}{3}\right) - \left(-1 - \frac{(-1)^3}{3}\right)\right] + \left[2(2) - 2(1)\right]$
$= 1 + \frac{2}{3} - \left(-\frac{2}{3}\right) + 4 - 2 = \frac{13}{3}$



16. $\int_{-1}^{2} h(r)\,dr = \int_{-1}^{0} r\,dr + \int_{0}^{1} (1-r^2)\,dr + \int_{1}^{2} dr$
$= \left[\frac{r^2}{2}\right]_{-1}^{0} + \left[r - \frac{r^3}{3}\right]_{0}^{1} + [r]_1^2$
$= \left(0 - \frac{(-1)^2}{2}\right) + \left(\left(1 - \frac{1^3}{3}\right) - 0\right) + (2-1)$
$= -\frac{1}{2} + \frac{2}{3} + 1 = \frac{7}{6}$



17. Ave. value $= \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{2-0}\int_0^2 f(x)\,dx = \frac{1}{2}\left[\int_0^1 x\,dx + \int_1^2 (x-1)\,dx\right] = \frac{1}{2}\left[\frac{x^2}{2}\right]_0^1 + \frac{1}{2}\left[\frac{x^2}{2} - x\right]_1^2$
$= \frac{1}{2}\left[\left(\frac{1^2}{2} - 0\right) + \left(\frac{2^2}{2} - 2\right) - \left(\frac{1^2}{2} - 1\right)\right] = \frac{1}{2}$

18. Ave. value $= \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{3-0}\int_0^3 f(x)\,dx = \frac{1}{3}\left[\int_0^1 dx + \int_1^2 0\,dx + \int_2^3 dx\right] = \frac{1}{3}[1 - 0 + 0 + 3 - 2] = \frac{2}{3}$

19. Let $f(x) = x^5$ on $[0,1]$. Partition $[0,1]$ into n subintervals with $\Delta x = \frac{1-0}{n} = \frac{1}{n}$. Then $\frac{1}{n}, \frac{2}{n}, \ldots, \frac{n}{n}$ are the right-hand endpoints of the subintervals. Since f is increasing on $[0,1]$, $U = \sum_{j=1}^{\infty} \left(\frac{j}{n}\right)^5 \left(\frac{1}{n}\right)$ is the upper sum for $f(x) = x^5$ on $[0,1]$ $\Rightarrow \lim_{n\to\infty} \sum_{j=1}^{\infty} \left(\frac{j}{n}\right)^5 \left(\frac{1}{n}\right) = \lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^5 + \left(\frac{2}{n}\right)^5 + \ldots + \left(\frac{n}{n}\right)^5\right] = \lim_{n\to\infty} \left[\frac{1^5 + 2^5 + \ldots + n^5}{n^6}\right]$
$= \int_0^1 x^5\,dx = \left[\frac{x^6}{6}\right]_0^1 = \frac{1}{6}$

20. Let $f(x) = x^3$ on $[0,1]$. Partition $[0,1]$ into n subintervals with $\Delta x = \frac{1-0}{n} = \frac{1}{n}$. Then $\frac{1}{n}, \frac{2}{n}, \ldots, \frac{n}{n}$ are the right-hand endpoints of the subintervals. Since f is increasing on $[0,1]$, $U = \sum_{j=1}^{\infty} \left(\frac{j}{n}\right)^3 \left(\frac{1}{n}\right)$ is the upper sum for $f(x) = x^3$ on $[0,1]$ $\Rightarrow \lim_{n\to\infty} \sum_{j=1}^{\infty} \left(\frac{j}{n}\right)^3 \left(\frac{1}{n}\right) = \lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^3 + \left(\frac{2}{n}\right)^3 + \ldots + \left(\frac{n}{n}\right)^3\right] = \lim_{n\to\infty} \left[\frac{1^3 + 2^3 + \ldots + n^3}{n^4}\right]$
$= \int_0^1 x^3\,dx = \left[\frac{x^4}{4}\right]_0^1 = \frac{1}{4}$

21. Let $y = f(x)$ on $[0,1]$. Partition $[0,1]$ into n subintervals with $\Delta x = \frac{1-0}{n} = \frac{1}{n}$. Then $\frac{1}{n}, \frac{2}{n}, \ldots, \frac{n}{n}$ are the right-hand endpoints of the subintervals. Since f is continuous on $[0,1]$, $\sum_{j=1}^{\infty} f\left(\frac{j}{n}\right)\left(\frac{1}{n}\right)$ is a Riemann sum of $y = f(x)$ on $[0,1]$ $\Rightarrow \lim_{n\to\infty} \sum_{j=1}^{\infty} f\left(\frac{j}{n}\right)\left(\frac{1}{n}\right) = \lim_{n\to\infty} \frac{1}{n}\left[f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \ldots + f\left(\frac{n}{n}\right)\right] = \int_0^1 f(x)\,dx$

22. (a) $\lim_{n\to\infty} \frac{1}{n^2}[2 + 4 + 6 + \ldots + 2n] = \lim_{n\to\infty} \frac{1}{n}\left[\frac{2}{n} + \frac{4}{n} + \frac{6}{n} + \ldots + \frac{2n}{n}\right] = \int_0^1 2x\,dx = \left[x^2\right]_0^1 = 1$, where $f(x) = 2x$ on $[0,1]$ (see Exercise 21)

(b) $\lim\limits_{n\to\infty} \frac{1}{n^{16}}\left[1^{15}+2^{15}+\ldots+n^{15}\right] = \lim\limits_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^{15}+\left(\frac{2}{n}\right)^{15}+\ldots+\left(\frac{n}{n}\right)^{15}\right] = \int_0^1 x^{15}\,dx = \left[\frac{x^{16}}{16}\right]_0^1 = \frac{1}{16}$, where $f(x)=x^{15}$ on $[0,1]$ (see Exercise 21)

(c) $\lim\limits_{n\to\infty} \frac{1}{n}\left[\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+\ldots+\sin\frac{n\pi}{n}\right] = \int_0^1 \sin n\pi\,dx = \left[-\frac{1}{\pi}\cos \pi x\right]_0^1 = -\frac{1}{\pi}\cos\pi - \left(-\frac{1}{\pi}\cos 0\right)$
$= \frac{2}{\pi}$, where $f(x)=\sin\pi x$ on $[0,1]$ (see Exercise 21)

(d) $\lim\limits_{n\to\infty} \frac{1}{n^{17}}\left[1^{15}+2^{15}+\ldots+n^{15}\right] = \left(\lim\limits_{n\to\infty}\frac{1}{n}\right)\left(\lim\limits_{n\to\infty}\frac{1}{n^{16}}\left[1^{15}+2^{15}+\ldots+n^{15}\right]\right) = \left(\lim\limits_{n\to\infty}\frac{1}{n}\right)\int_0^1 x^{15}\,dx$
$= 0\left(\frac{1}{16}\right) = 0$ (see part (b) above)

(e) $\lim\limits_{n\to\infty} \frac{1}{n^{15}}\left[1^{15}+2^{15}+\ldots+n^{15}\right] = \lim\limits_{n\to\infty} \frac{n}{n^{16}}\left[1^{15}+2^{15}+\ldots+n^{15}\right]$
$= \left(\lim\limits_{n\to\infty} n\right)\left(\lim\limits_{n\to\infty}\frac{1}{n^{16}}\left[1^{15}+2^{15}+\ldots+n^{15}\right]\right) = \left(\lim\limits_{n\to\infty} n\right)\int_0^1 x^{15}\,dx = \infty$ (see part (b) above)

23. (a) Let the polygon be inscribed in a circle of radius r. If we draw a radius from the center of the circle (and the polygon) to each vertex of the polygon, we have n isosceles triangles formed (the equal sides are equal to r, the radius of the circle) and a vertex angle of θ_n where $\theta_n = \frac{2\pi}{n}$. The area of each triangle is $A_n = \frac{1}{2}r^2\sin\theta_n \Rightarrow$ the area of the polygon is $A = nA_n = \frac{nr^2}{2}\sin\theta_n = \frac{nr^2}{2}\sin\frac{2\pi}{n}$.

(b) $\lim\limits_{n\to\infty} A = \lim\limits_{n\to\infty}\frac{nr^2}{2}\sin\frac{2\pi}{n} = \lim\limits_{n\to\infty}\frac{n\pi r^2}{2\pi}\sin\frac{2\pi}{n} = \lim\limits_{n\to\infty}(\pi r^2)\frac{\sin\left(\frac{2\pi}{n}\right)}{\left(\frac{2\pi}{n}\right)} = (\pi r^2)\lim\limits_{2\pi/n\to 0}\frac{\sin\left(\frac{2\pi}{n}\right)}{\left(\frac{2\pi}{n}\right)} = \pi r^2$

24. Partition $[0,1]$ into n subintervals, each of length $\Delta x = \frac{1}{n}$ with the points $x_0 = 0$, $x_1 = \frac{1}{n}$, $x_2 = \frac{2}{n}, \ldots, x_n = \frac{n}{n} = 1$. The inscribed rectangles so determined have areas
$f(x_0)\,\Delta x = (0)^2\,\Delta x$, $f(x_1)\,\Delta x = \left(\frac{1}{n}\right)^2\Delta x$, $f(x_2)\,\Delta x = \left(\frac{2}{n}\right)^2\Delta x, \ldots, f(x_{n-1}) = \left(\frac{n-1}{n}\right)^2\Delta x$. The sum of these areas is $S_n = \left(0^2+\left(\frac{1}{n}\right)^2+\left(\frac{2}{n}\right)^2+\ldots+\left(\frac{n-1}{n}\right)^2\right)\Delta x = \left(\frac{1^2}{n^2}+\frac{2^2}{n^2}+\ldots+\frac{(n-1)^2}{n^2}\right)\frac{1}{n} = \frac{1^2}{n^3}+\frac{2^2}{n^3}+\ldots+\frac{(n-1)^2}{n^3}$. Then $\lim\limits_{n\to\infty} S_n = \lim\limits_{n\to\infty}\left(\frac{1^2}{n^3}+\frac{2^2}{n^3}+\ldots+\frac{(n-1)^2}{n^3}\right) = \int_0^1 x^2\,dx = \frac{1^3}{3} = \frac{1}{3}$.

25. (a) $g(1) = \int_1^1 f(t)\,dt = 0$

(b) $g(3) = \int_1^3 f(t)\,dt = -\frac{1}{2}(2)(1) = -1$

(c) $g(-1) = \int_1^{-1} f(t)\,dt = -\int_{-1}^1 f(t)\,dt = -\frac{1}{4}(\pi\,2^2) = -\pi$

(d) $g'(x) = f(x) = 0 \Rightarrow x = -3, 1, 3$ and the sign chart for $g'(x) = f(x)$ is $\underset{-3}{|}+++\underset{1}{|}---\underset{3}{|}+++$. So g has a relative maximum at $x = 1$.

(e) $g'(-1) = f(-1) = 2$ is the slope and $g(-1) = \int_1^{-1} f(t)\,dt = -\pi$, by (c). Thus the equation is $y + \pi = 2(x+1)$
$y = 2x + 2 - \pi$.

(f) $g''(x) = f'(x) = 0$ at $x = -1$ and $g''(x) = f'(x)$ is negative on $(-3,\,-1)$ and positive on $(-1,\,1)$ so there is an inflection point for g at $x = -1$. We notice that $g''(x) = f'(x) < 0$ for x on $(-1,\,2)$ and $g''(x) = f'(x) > 0$ for x on $(2,\,4)$, even though $g''(2)$ does not exist, g has a tangent line at $x = 2$, so there is an inflection point at $x = 2$.

(g) g is continuous on $[-3,\,4]$ and so it attains its absolute maximum and minimum values on this interval. We saw in (d) that $g'(x) = 0 \Rightarrow x = -3, 1, 3$. We have that

$g(-3) = \int_1^{-3} f(t)\,dt = -\int_{-3}^1 f(t)\,dt = -\frac{\pi\,2^2}{2} = -2\pi$

$g(1) = \int_1^1 f(t)\,dt = 0$

$g(3) = \int_1^3 f(t)\,dt = -1$

$g(4) = \int_1^4 f(t)\,dt = -1 + \frac{1}{2}\cdot 1\cdot 1 = -\frac{1}{2}$

Thus, the absolute minimum is -2π and the absolute maximum is 0. Thus, the range is $[-2\pi,\,0]$.

26. $y = \sin x + \int_x^{\pi} \cos 2t\, dt + 1 = \sin x - \int_{\pi}^{x} \cos 2t\, dt + 1 \Rightarrow y' = \cos x - \cos(2x)$; when $x = \pi$ we have $y' = \cos \pi - \cos(2\pi) = -1 - 1 = -2$. And $y'' = -\sin x + 2\sin(2x)$; when $x = \pi$, $y = \sin \pi + \int_x^{\pi} \cos 2t\, dt + 1$ $= 0 + 0 + 1 = 1$.

27. $f(x) = \int_{1/x}^{x} \frac{1}{t}\, dt \Rightarrow f'(x) = \frac{1}{x}\left(\frac{dx}{dx}\right) - \left(\frac{1}{\frac{1}{x}}\right)\left(\frac{d}{dx}\left(\frac{1}{x}\right)\right) = \frac{1}{x} - x\left(-\frac{1}{x^2}\right) = \frac{1}{x} + \frac{1}{x} = \frac{2}{x}$

28. $f(x) = \int_{\cos x}^{\sin x} \frac{1}{1-t^2}\, dt \Rightarrow f'(x) = \left(\frac{1}{1-\sin^2 x}\right)\left(\frac{d}{dx}(\sin x)\right) - \left(\frac{1}{1-\cos^2 x}\right)\left(\frac{d}{dx}(\cos x)\right) = \frac{\cos x}{\cos^2 x} + \frac{\sin x}{\sin^2 x}$ $= \frac{1}{\cos x} + \frac{1}{\sin x}$

29. $g(y) = \int_{\sqrt{y}}^{2\sqrt{y}} \sin t^2\, dt \Rightarrow g'(y) = \left(\sin\left(2\sqrt{y}\right)^2\right)\left(\frac{d}{dy}\left(2\sqrt{y}\right)\right) - \left(\sin\left(\sqrt{y}\right)^2\right)\left(\frac{d}{dy}\left(\sqrt{y}\right)\right) = \frac{\sin 4y}{\sqrt{y}} - \frac{\sin y}{2\sqrt{y}}$

30. $f(x) = \int_x^{x+3} t(5-t)\, dt \Rightarrow f'(x) = (x+3)(5-(x+3))\left(\frac{d}{dx}(x+3)\right) - x(5-x)\left(\frac{dx}{dx}\right) = (x+3)(2-x) - x(5-x)$ $= 6 - x - x^2 - 5x + x^2 = 6 - 6x$. Thus $f'(x) = 0 \Rightarrow 6 - 6x = 0 \Rightarrow x = 1$. Also, $f''(x) = -6 < 0 \Rightarrow x = 1$ gives a maximum.

NOTES